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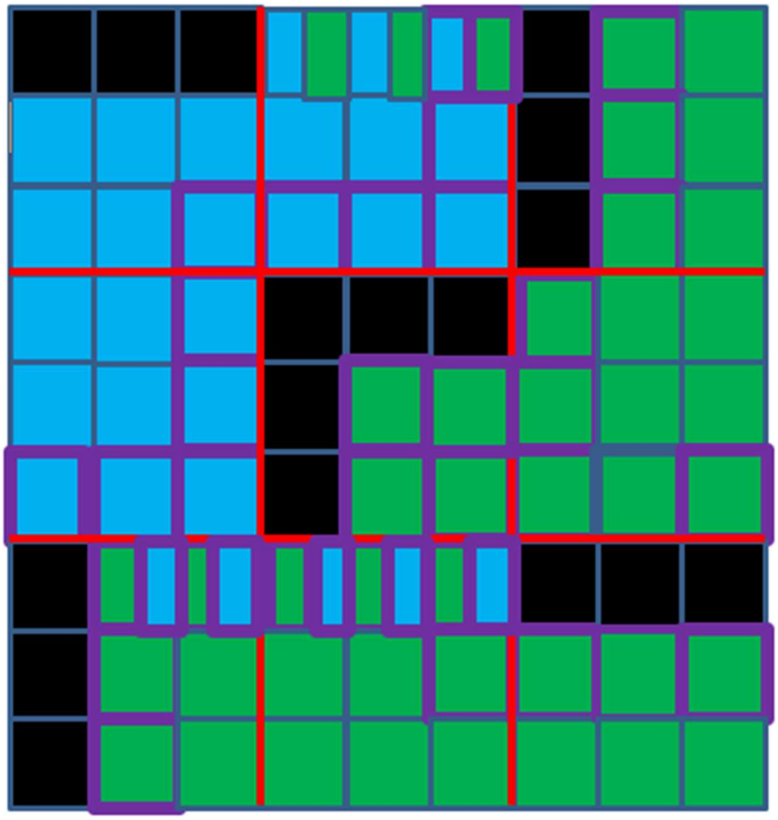
Thematisierungstheorie



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Vorwort

Im Anschluß an Benses Semiotik kann mindestens zwischen den folgenden semiotischen Teiltheorien unterschieden werden: Regelungstheorie, Invariantentheorie, Fundierungstheorie, Valenztheorie, Pfadtheorie, Kreationstheorie, Selektionstheorie, Substitutionstheorie, Transformationstheorie. Hinzu kommt als weitere die im vorliegenden Bande in ihren verschiedenen Aspekten dargestellte Thematisierungstheorie. Von Thematisierung wird in der Semiotik im Zusammenhang mit den von den Realitätsthematiken thematisierten strukturellen oder entitätischen Realitäten gesprochen. Bisher nie als Einzeltheorie herausgearbeitet, gehörte die Thematisierungstheorie bereits anfangs der 70er Jahre, wie aus dem folgenden Zitat aus dem "Wörterbuch der Semiotik" hervorgeht, zu den zentralen semiotischen Konzepten: "Die thematisierende und generierende, die repräsentierende, kategorisierende und relationierende Leistung der Zeichen ist ebenso eine Folge ihrer Metaobjekt-Natur wie ihre modale Charakteristik als (trägergebundene) Mitrealität".

Mit dem vorliegenden Bande sind im Rahmen der Monographien des „Semiotic Technical Laboratories“ nunmehr sämtlich semiotischen Teiltheorien behandelt. Die in diesem Buche versammelten Aufsätze stammen aus der Zeit von 2007 bis 2020.

Tucson, AZ, 20.2.2020 Prof. Dr. Alfred Toth

Tetradic sign classes from relational and categorial numbers

1. In Toth (2008b), we had elaborated Bense's introduction of relational and categorial numbers in order to fully characterize sign relations Zr_k (Bense 1975, pp. 65 s.). Zr_k includes pre-semiotic media relations (M°) which connect Zr_k as a representation scheme of the semiotic space with the ontological space out of which objects are selected to be thetically introduced as meta-objects and thus as signs (Bense 1967, p. 9). This distinction allows to differentiating between the semiotic sign relation

SR = (.1., .2., .3.)

and the pre-semiotic qualitative-quantitative sign relation

PSR = (0., .1., .2., .3.).

Since, in Zr_k, $k \neq 0$, the respective pre-semiotic matrix does not contain the zeroness in trichotomic position. Hence the pre-semiotic matrix is "defective" from the viewpoint of a total-symmetric matrix of Cartesian products over (.0., .1., .2., .3.):

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

From that it follows, too, that sign classes built from the 12 sub-signs in the pre-semiotic matrix will not lead to the system of the 35 tetradic-tetratomic sign classes shown and discussed in Toth (2008a, pp. 179 ss.). If we apply the trichotomic semiotic order in triadic semiotic sign classes:

(3.a 2.b 1.c) with $a \leq b \leq c$

to the tetratomic order in tetradic pre-semiotic sign classes:

(3.a 2.b 1.c 0.d) with $a \leq b \leq c \leq d$,

then we can construct the following 15 pre-semiotic sign classes:

1 (3.1 2.1 1.1 0.1) × ($\underline{1.0\ 1.1\ 1.2\ 1.3}$)
2 (3.1 2.1 1.1 0.2) × (2.0 $\underline{1.1\ 1.2\ 1.3}$)
3 (3.1 2.1 1.1 0.3) × (3.0 $\underline{1.1\ 1.2\ 1.3}$)
4 (3.1 2.1 1.2 0.2) × ($\underline{2.0\ 2.1}$ $\underline{1.2\ 1.3}$)
5 (3.1 2.1 1.2 0.3) × (3.0 2.1 $\underline{1.2\ 1.3}$)
6 (3.1 2.1 1.3 0.3) × ($\underline{3.0\ 3.1}$ $\underline{1.2\ 1.3}$)

7 (3.1 2.2 1.2 0.2) × ($\underline{2.0\ 2.1\ 2.2}$ 1.3)
8 (3.1 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2}$ 1.3)
9 (3.1 2.2 1.3 0.3) × ($\underline{3.0\ 3.1}$ 2.2 1.3)
10 (3.1 2.3 1.3 0.3) × ($\underline{3.0\ 3.1\ 3.2}$ 1.3)
11 (3.2 2.2 1.2 0.2) × ($\underline{2.0\ 2.1\ 2.2\ 2.3}$)
12 (3.2 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2\ 2.3}$)
13 (3.2 2.2 1.3 0.3) × ($\underline{3.0\ 3.1}$ $\underline{2.2\ 2.3}$)
14 (3.2 2.3 1.3 0.3) × ($\underline{3.0\ 3.1\ 3.2}$ 2.3)
15 (3.3 2.3 1.3 0.3) × ($\underline{3.0\ 3.1\ 3.2\ 3.3}$),

whose number corresponds to the 15 trito-numbers of the polycontextural contexture T_4 (cf. Kronthaler 1986, p. 34), which underlines the fact that these 15 pre-semiotic sign classes are both quantitative and qualitative sign classes, because the integration of the zeroness into the triadic sign relation bridges the polycontextural border between the ontological space of objects and the semiotic space of signs (cf. Toth 2003, 2008a).

Moreover, we notice that in the system of the 15 pre-semiotic classes, there is, unlike in the system of the 10 semiotic sign classes, no dual-identical sign class corresponding to the triadic "eigenreal" sign class (3.1 2.2 1.3) × (3.1 2.2 1.3), cf. Bense (1992). On the other side, the system of the 15 pre-semiotic sign classes displays, in the system of their dual reality thematics, semiotic structures that do not occur in the system of the 10 semiotic sign classes. In order to "formalize" them, we use the notational system introduced in Toth (2008a, pp. 176 ss.). The abbreviation HOM stands for homogeneous thematizations, LEFT and RIGHT refer to the direction of thematizations (indicated by arrows), and SWCH for "sandwich thematization" points to the fact that in the respective structural realities two realities are thematizing and two are thematized. Then we get the following types of tetradic thematizations of the 15 pre-semiotic sign classes:

1. Homogeneous thematizations:

1	(3.1 2.1 1.1 0.1) × ($\underline{1.0\ 1.1\ 1.2\ 1.3}$)	1^4	HOM
11	(3.2 2.2 1.2 0.2) × ($\underline{2.0\ 2.1\ 2.2\ 2.3}$)	2^4	HOM
15	(3.3 2.3 1.3 0.3) × ($\underline{3.0\ 3.1\ 3.2\ 3.3}$)	3^4	HOM

2. Dyadic thematizations

2.1. Dyadic-leftward thematizations

2	(3.1 2.1 1.1 0.2) × (2.0 $\underline{1.1\ 1.2\ 1.3}$)	$2^1 \leftarrow 1^3$	LEFT
3	(3.1 2.1 1.1 0.3) × (3.0 $\underline{1.1\ 1.2\ 1.3}$)	$3^1 \leftarrow 1^3$	LEFT
12	(3.2 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2\ 2.3}$)	$3^1 \leftarrow 2^3$	LEFT

2.2. Dyadic-rightward thematizations

7	$(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$	$2^3 \rightarrow 1^1$	RIGHT
10	$(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$	$3^3 \rightarrow 1^1$	RIGHT
14	$(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$	$3^3 \rightarrow 2^1$	RIGHT

2.3. Sandwich-Thematizations (only centripetal)

4	$(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$	$2^2 \leftrightarrow 1^2$	SWCH
6	$(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$	$3^2 \leftrightarrow 1^2$	SWCH
13	$(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$	$3^2 \leftrightarrow 2^2$	SWCH

3. Triadic thematizations

3.1. Triadic-leftward thematization

5	$(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$	$3^1 \leftrightarrow 2^1 \leftarrow 1^2$	LEFT

3.2. Triadic-rightward thematization

9	$(3.1\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ 2.2\ 1.3)$	$3^2 \rightarrow 2^1 \leftrightarrow 1^3$	RIGHT

3.3. Sandwich-thematization (centrifugal)

8	$(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$	$3^1 \leftarrow 2^2 \rightarrow 1^1$	SWCH

It is easy to recognize that the 15 reality thematics of the system of the tetradic pre-semiotic sign classes can not be organized into a system of tetratomic tetrads analogous to the system of trichotomic triads (cf. Walther 1982). The latter is symmetric by aid of the determinant of the eigenreal sign class (3.1 2.2 1.3), and since there is no eigenreality in the system of the 15 pre-semiotic sign classes, they can not be constructed as n-adic m-ary semiotic systems in which n = m like in the case of the tetratomic tetrads constructed out of the 35 tetradic-tetratomic sign classes in Toth (2008a, pp. 180 ss.).

However, it is possible to construct a system of **triadic pentatomies** out of the system of the 15 pre-semiotic sign classes:

1	$(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$	1^4	HOM
2	$(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$	$2^1 \leftarrow 1^3$	LEFT
4	$(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$	$2^2 \leftrightarrow 1^2$	SWCH
7	$(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$	$2^3 \rightarrow 1^1$	RIGHT
5	$(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$	$3^1 \leftrightarrow 2^1 \leftarrow 1^2$	LEFT

11	$(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$	2^4	HOM
3	$(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$	$3^1 \leftarrow 1^3$	LEFT
6	$(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$	$3^2 \leftrightarrow 1^2$	SWCH
10	$(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$	$3^3 \rightarrow 1^1$	RIGHT
9	$(3.1\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ 2.2\ 1.3)$	$3^2 \rightarrow 2^1 \leftrightarrow 1^3$	RIGHT

15	$(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$	3^4	HOM
12	$(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2\ 2.3})$	$3^1 \leftarrow 2^3$	LEFT
13	$(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$	$3^2 \leftrightarrow 2^2$	SWCH
14	$(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$	$3^3 \rightarrow 2^1$	RIGHT
8	$(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$	$3^1 \leftarrow 2^2 \rightarrow 1^1$	SWCH

We recognize that each of these pentatomies has the following structure. X, Y, and Z $\in$ {1, 2, 3}:

X^4	HOM
$X^1 \leftarrow Y^3$	LEFT
$X^2 \leftrightarrow Y^2$	SWCH
$X^3 \rightarrow Y^1$	RIGHT
$X^1 \leftarrow X^2 \rightarrow Z^1$	SWCH

Thus, although the structural realities presented in the tetratomic reality thematics are tetradic, zeroness appears as triadic sign value and thus in the sign classes, but not as tetradic value and thus not in the reality thematics. In other words: In order to describe the realities presented by the tetradic pre-semiotic sign classes, **three** semiotic categories (X, Y, Z) are sufficient. Therefore, according to Bense (1975, pp. 64 ss. and Toth 2008b), the X, Y, Z refer to the **categorial numbers**, and the “exponents” in the above frequency notation of structural realities refer to the **relational numbers**. Using this frequency notation, we are able, on the basis of the above pentatomic structure of tetradic realities, to construct the system of the triadic pentatomies from the system of the 15 pre-semiotic sign classes based on the pre-semiotic sign relation PSR = (3.a 2.b 1.c 0.d), the tetratomic pre-semiotic order ($a \leq b \leq c \leq d$) and the restriction that zeroness must not appear in trichotomic position.

This n-adic m-ary semiotic system for n = 3 and m = 5 thus connects by its n-adic value the pre-semiotic system of the 15 sign classes with the triadic system of the 10 sign classes which therefore appear as a morphogrammatic fragment of the system of the 15 pre-semiotic sign classes, on the one side, and with a pentadic-m-ary system of ≤ 126 sign classes (cf. Toth 2008a, pp. 186 ss.) whose fragment the system of the 15 pre-semiotic sign classes is, on the other side (cf. Toth 2003, pp. 54 ss.).

Finally, one should notice that the absence of a dual-identical sign class in order to express eigenreality in the system of the 15 pre-semiotic sign classes leads to the fact that these pre-semiotic sign classes cannot be dualized, but must be triadized (cf. Kronthaler 1992, p. 293). Triadization is thus the minimal condition to transform one of the 15 pre-semiotic sign classes by reversing both the order of its dyadic sub-relations and of its monadic prime-signs back to its original sign class structure:

6 (3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3) × (3.1 2.2 1.2 0.3),

The following study is the first contribution to **Pre-semiotics** in the sense of the theory of the pre-semiotic sign classes, their **trial** reality thematics and their associated system of triadic pentatomies. The main aim of Pre-semiotics is to formally analyze and describe the “never-land” between the Ontological and the Semiotic Space in the sense of Bense (1975, p. 65) and to disclose the pre-semiotic relations in the network of the abyss between sign and object.

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Tetradic, triadic, and dyadic sign classes

1. In Toth (2008a, pp. 179 ss.), we have constructed a tetradic-tetratomic semiotics on the basis of the following 4×4 matrix:

	.0	.1	.2	.3
0.	0.0	0.1	0.2	0.3
1.	1.0	1.1	1.2	1.3
2.	2.0	2.1	2.2	2.3
3.	3.0	3.1	3.2	3.3

based on the general tetradic-tetratomic sign relation

SR_4 = R(Q, M, O, I); SR_4 = R(.0., .1., .2., .3.);

SR_4 = (((Q $\Rightarrow$ M) $\Rightarrow$ O) $\Rightarrow$ I); SR_4 = (((.0. $\Rightarrow$ .1.) $\Rightarrow$ .2.) $\Rightarrow$ .3.)

with the tetratomic semiotic inclusion order

(3.a 2.b 1.c 0.d) with a, b, c, d $\in$ {.0., .1., .2., .3.} und $a \leq b \leq c \leq d$

We can then construct the following 35 tetradic-tetratomic sign classes and their dual reality thematics:

1	(3.0 2.0 1.0 0.0)	$\times$	(0.0 0.1 0.2 0.3)
2	(3.0 2.0 1.0 0.1)	$\times$	(1.0 0.1 0.2 0.3)
3	(3.0 2.0 1.0 0.2)	$\times$	(2.0 0.1 0.2 0.3)
4	(3.0 2.0 1.0 0.3)	$\times$	(3.0 0.1 0.2 0.3)
5	(3.0 2.0 1.1 0.1)	$\times$	(1.0 1.1 0.2 0.3)
6	(3.0 2.0 1.1 0.2)	$\times$	(2.0 1.1 0.2 0.3)
7	(3.0 2.0 1.1 0.3)	$\times$	(3.0 1.1 0.2 0.3)
8	(3.0 2.0 1.2 0.2)	$\times$	(2.0 2.1 0.2 0.3)
9	(3.0 2.0 1.2 0.3)	$\times$	(3.0 2.1 0.2 0.3)
10	(3.0 2.0 1.3 0.3)	$\times$	(3.0 3.1 0.2 0.3)
11	(3.0 2.1 1.1 0.1)	$\times$	(1.0 1.1 1.2 0.3)
12	(3.0 2.1 1.1 0.2)	$\times$	(2.0 1.1 1.2 0.3)

13	(3.0	2.1	1.1	0.3)	×	(3.0	1.1	1.2	0.3)
14	(3.0	2.1	1.2	0.2)	×	(2.0	2.1	1.2	0.3)
15	(3.0	2.1	1.2	0.3)	×	(3.0	2.1	1.2	0.3)
16	(3.0	2.1	1.3	0.3)	×	(3.0	3.1	1.2	0.3)
17	(3.0	2.2	1.2	0.2)	×	(2.0	2.1	2.2	0.3)
18	(3.0	2.2	1.2	0.3)	×	(3.0	2.1	2.2	0.3)
19	(3.0	2.2	1.3	0.3)	×	(3.0	3.1	2.2	0.3)
20	(3.0	2.3	1.3	0.3)	×	(3.0	3.1	3.2	0.3)
21	(3.1	2.1	1.1	0.1)	×	(1.0	1.1	1.2	1.3)
22	(3.1	2.1	1.1	0.2)	×	(2.0	1.1	1.2	1.3)
23	(3.1	2.1	1.1	0.3)	×	(3.0	1.1	1.2	1.3)
24	(3.1	2.1	1.2	0.2)	×	(2.0	2.1	1.2	1.3)
25	(3.1	2.1	1.2	0.3)	×	(3.0	2.1	1.2	1.3)
26	(3.1	2.1	1.3	0.3)	×	(3.0	3.1	1.2	1.3)
27	(3.1	2.2	1.2	0.2)	×	(2.0	2.1	2.2	1.3)
28	(3.1	2.2	1.2	0.3)	×	(3.0	2.1	2.2	1.3)
29	(3.1	2.2	1.3	0.3)	×	(3.0	3.1	2.2	1.3)
30	(3.1	2.3	1.3	0.3)	×	(3.0	3.1	3.2	1.3)
31	(3.2	2.2	1.2	0.2)	×	(2.0	2.1	2.2	2.3)
32	(3.2	2.2	1.2	0.3)	×	(3.0	2.1	2.2	2.3)
33	(3.2	2.2	1.3	0.3)	×	(3.0	3.1	2.2	2.3)
34	(3.2	2.3	1.3	0.3)	×	(3.0	3.1	3.2	2.3)
35	(3.3	2.3	1.3	0.3)	×	(3.0	3.1	3.2	3.3)

The 35 representation systems can be ordered into the following system of **4 Tetratomic Tetrads of structural realities with dyadic thematization**:

1	(3.0	2.0	1.0	0.0)	×	(0.0	0.1	0.2	0.3)
2	(3.0	2.0	1.0	0.1)	×	(1.0	0.1	0.2	0.3)
3	(3.0	2.0	1.0	0.2)	×	(2.0	0.1	0.2	0.3)

4	(3.0 2.0 1.0 0.3)	×	$(3.0\ \underline{0.1\ \ 0.2\ \ 0.3})$

11	(3.0 2.1 1.1 0.1)	×	$(\underline{1.0\ \ 1.1\ \ 1.2}\ \ 0.3)$
21	(3.1 2.1 1.1 0.1)	×	$(\underline{1.0\ \ 1.1\ \ 1.2\ \ 1.3})$
22	(3.1 2.1 1.1 0.2)	×	$(2.0\ \ \underline{1.1\ \ 1.2\ \ 1.3})$
23	(3.1 2.1 1.1 0.3)	×	$(3.0\ \ \underline{1.1\ \ 1.2\ \ 1.3})$

17	(3.0 2.2 1.2 0.2)	×	$(\underline{2.0\ \ 2.1\ \ 2.2}\ \ 0.3)$
27	(3.1 2.2 1.2 0.2)	×	$(\underline{2.0\ \ 2.1\ \ 2.2}\ \ 1.3)$
31	(3.2 2.2 1.2 0.2)	×	$(\underline{2.0\ \ 2.1\ \ 2.2\ \ 2.3})$
32	(3.2 2.2 1.2 0.3)	×	$(3.0\ \ \underline{2.1\ \ 2.2\ \ 2.3})$

20	(3.0 2.3 1.3 0.3)	×	$(\underline{3.0\ \ 3.1\ \ 3.2}\ \ 0.3)$
30	(3.1 2.3 1.3 0.3)	×	$(\underline{3.0\ \ 3.1\ \ 3.2}\ \ 1.3)$
34	(3.2 2.3 1.3 0.3)	×	$(\underline{3.0\ \ 3.1\ \ 3.2}\ \ 2.3)$
35	(3.3 2.3 1.3 0.3)	×	$(\underline{3.0\ \ 3.1\ \ 3.2\ \ 3.3})$

Moreover, the 35 representation systems can also be ordered into the following system **of 4 Tetratomic Triads of triadic thematization**:

1	(3.0 2.0 1.0 0.0)	×	$(\underline{0.0\ \ 0.1\ \ 0.2\ \ 0.3})$
6	(3.0 2.0 1.1 0.2)	×	$(2.0\ \ 1.1\ \ \underline{0.2\ \ 0.3})$
9	(3.0 2.0 1.2 0.3)	×	$(3.0\ \ 2.1\ \ \underline{0.2\ \ 0.3})$
7	(3.0 2.0 1.1 0.3)	×	$(3.0\ \ 1.1\ \ \underline{0.2\ \ 0.3})$

12	(3.0 2.1 1.1 0.2)	×	$(2.0\ \ \underline{1.1\ \ 1.2}\ \ 0.3)$
21	(3.1 2.1 1.1 0.1)	×	$(\underline{1.0\ \ 1.1\ \ 1.2\ \ 1.3})$
25	(3.1 2.1 1.2 0.3)	×	$(3.0\ \ 2.1\ \ \underline{1.2\ \ 1.3})$
13	(3.0 2.1 1.1 0.3)	×	$(3.0\ \ \underline{1.1\ \ 1.2}\ \ 0.3)$

14	(3.0	2.1	1.2	0.2)	×	(2.0	2.1	1.2	0.3)
28	(3.1	2.2	1.2	0.3)	×	(3.0	2.1	2.2	1.3)
31	(3.2	2.2	1.2	0.2)	×	(2.0	2.1	2.2	2.3)
18	(3.0	2.2	1.2	0.3)	×	(3.0	2.1	2.2	0.3)

16	(3.0	2.1	1.3	0.3)	×	(3.0	3.1	1.2	0.3)
29	(3.1	2.2	1.3	0.3)	×	(3.0	3.1	2.2	1.3)
19	(3.0	2.2	1.3	0.3)	×	(3.0	3.1	2.2	0.3)
35	(3.3	2.3	1.3	0.3)	×	(3.0	3.1	3.2	3.3)

2. Triadic-trichotomic semiotics that is constructed by aid of the following 3 × 3 matrix:

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

on the basis of the general triadic-trichotomic sign relation

SR_3 = R(M, O, I); SR_3 = R(.1., .2., .3.);

SR_3 = ((M $\Rightarrow$ O) $\Rightarrow$ I); SR_3 = ((.1. $\Rightarrow$ .2.) $\Rightarrow$ .3.)

with the trichotomic semiotic inclusion order

(3.a 2.b 1.c) with a, b, c $\in$ {.1., .2., .3.} und a $\leq$ b $\leq$ c

has the following 10 triadic-trichotomic sign classes and their dual reality thematics:

1	(3.1	2.1	1.1)	×	(1.1	1.2	1.3)
2	(3.1	2.1	1.2)	×	(2.1	1.2	1.3)
3	(3.1	2.1	1.3)	×	(3.1	1.2	1.3)
4	(3.1	2.2	1.2)	×	(2.1	2.2	1.3)
5	(3.1	2.2	1.3)	×	(3.1	2.2	1.3)
6	(3.1	2.3	1.3)	×	(3.1	3.2	1.3)
7	(3.2	2.2	1.2)	×	(2.1	2.2	2.3)
8	(3.2	2.2	1.3)	×	(3.1	2.2	2.3)
9	(3.2	2.3	1.3)	×	(3.1	3.2	2.3)
10	(3.3	2.3	1.3)	×	(3.1	3.2	3.3)

The 10 representation systems can be ordered into the following system of **3 Trichotomic Triads** (Walther 1981, 1982):

$$\begin{array}{llll}
1 & (3.1\ \ 2.1\ \ 1.1) & \times & (\underline{1.1\ \ 1.2\ \ 1.3}) \\
2 & (3.1\ \ 2.1\ \ 1.2) & \times & (2.1\ \ \underline{1.2\ \ 1.3}) \\
3 & (3.1\ \ 2.1\ \ 1.3) & \times & (3.1\ \ \underline{1.2\ \ 1.3}) \\
 & & & \\
4 & (3.1\ \ 2.2\ \ 1.2) & \times & (\underline{2.1\ \ 2.2}\ \ 1.3) \\
7 & (3.2\ \ 2.2\ \ 1.2) & \times & (\underline{2.1\ \ 2.2\ \ 2.3}) \\
8 & (3.2\ \ 2.2\ \ 1.3) & \times & (3.1\ \ \underline{2.2\ \ 2.3}) \\
 & & & \\
6 & (3.1\ \ 2.3\ \ 1.3) & \times & (\underline{3.1\ \ 3.2}\ \ 1.3) \\
9 & (3.2\ \ 2.3\ \ 1.3) & \times & (\underline{3.1\ \ 3.2}\ \ 2.3) \\
10 & (3.3\ \ 2.3\ \ 1.3) & \times & (\underline{3.1\ \ 3.2\ \ 3.3})
\end{array}$$

Here, the dual-invariant sign class (3.1 2.2 1.3) × (3.1 2.2 1.3), the determinant of the triadic-trichotomic matrix, determines the system of the Trichotomic Triads. In the 2 systems of the 35 tetradic sign classes, the dual-invariant sign class (3.0 2.1 1.2 0.3) × (3.0 2.1 1.2 0.3), the determinant of the tetradic-tetratomic matrix, determines the 2 systems of the Tetratomic Tetrads. While (3.1 2.2 1.3) has the following three types of thematizations and thus structural realities:

$$(3.1\ 2.2\ 1.3) \times (\underline{3.1}\ \underline{2.1}\ \underline{1.3}) \rightarrow \begin{cases} (3.1, 2.1)\text{-them. } (1.3) \\ (3.1, 1.3)\text{-them. } (2.2) \\ (2.2, 1.3)\text{-them. } (3.1), \end{cases}$$

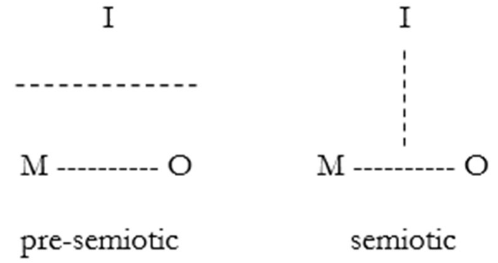
the sign class (3.0 2.1 1.2 0.3) has 10 types of thematizations and structural realities (thematized realities are underlined):

$$(3.0\ 2.1\ 1.2\ 0.3) \times (\underline{3.0}\ \underline{2.1}\ \underline{1.2}\ 0.3) \quad \rightarrow \quad \begin{array}{l}
(\underline{3.0\ \ 2.1\ \ 1.2}\ \ 0.3) \\
(\underline{3.0\ \ 2.1}\ \ 1.2\ \ 0.3) \\
(\underline{3.0}\ \ 2.1\ \ 1.2\ \ 0.3) \\
(3.0\ \ \underline{2.1\ \ 1.2\ \ 0.3}) \\
(3.0\ \ 2.1\ \ \underline{1.2\ \ 0.3}) \\
(3.0\ \ 2.1\ \ 1.2\ \ \underline{0.3}) \\
(\underline{3.0}\ \ 2.1\ \ 1.2\ \ \underline{0.3}) \\
(3.0\ \ \underline{2.1\ \ 1.2}\ \ 0.3) \\
(3.0\ \ \underline{2.1}\ \ 1.2\ \ 0.3) \\
(3.0\ \ 2.1\ \ \underline{1.2}\ \ 0.3)
\end{array}$$

Thus, from their structural realities and from their possibilities to be ordered into a system of n-atomic n-ads, SR_3 is **not** a part of SR_4, since SR_4 has quite different n-adic n-atomic and thematization structures than SR_3.

3. Ditterich (1990, pp. 29, 81) has defined the dyadic sign relation of de Saussure, which he calls „pre-semiotic“, by aid of the semiotic matrix as a sub-relation of the triadic-trichotomic Peircean sign relation SR_3:

	.1	.2	.3
3.	3.1	3.2	3.3
2.	2.1	2.2	2.3
1.	1.1	1.2	1.3



$(1 \rightarrow 2) \,/ \rightarrow 3$ $\qquad$ $((1 \rightarrow 2). \rightarrow 3)$

If we write the dyadic sign relation as SR_2, then we have according to Ditterich:

$SR_2 \subset SR_3$,

However, it is not clear, if this inclusion holds beyond the pure quantitative point of view. In the triadic sign model, the third category, the interpretant or the thirdness, alone guarantees that the triadic sign is a “mediating function between World and Consiousness” (Bense 1975, p. 16; 1976, p. 91; Toth 2008b). Thus, if the interpretant relation falls off, the sign cannot mediate anymore between the dyadic rest-function and the consciousness of the interpreter. Therefore, the interpretant relation which embeds the dyadic relation $(M \Rightarrow O)$ into the triadic relation $((M \Rightarrow O) \Rightarrow I)$ crosses the contexture of the denomination function $(M \Rightarrow O)$ that belongs to the “world” and adds to it the designation function $(O \Rightarrow I)$ that belongs to the “consciousness”. Hence, already the triadic sign relation involves two logical contextures, world and consciousness, or object and subject that are bridged in the triadic sign relation. From that it follows, that Ditterich’s inclusion relation does not hold from the qualitative point of view (cf. also Toth 1991), so that we have

$SR_2 \not\subset SR_3$.

4. In Toth (2008c), I have introduced the tetradic-trichotomic pre-semiotic sign relation

$PSR = (0., .1., .2., .3.); SR_{4,3}$ (3.a 2.b 1.c 0.d)

with the corresponding trichotomic inclusion order

$(a \leq b \leq c)$,

whose corresponding semiotic structure is thus 4-adic, but 3-ary, since in Zr_k, the categorial number $k \neq 0$ (Bense 1975, p. 65), and therefore the pre-semiotic matrix is “defective” from the viewpoint of a quadratic matrix of Cartesian products over (.0., .1., .2., .3.):

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

From this semiotic matrix, we can construct the following 15 tetradic-trichotomic sign classes and their dual reality thematics:

1 $(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$
2 $(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$
3 $(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$
4 $(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$
5 $(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$
6 $(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$
7 $(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$
8 $(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$
9 $(3.1\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ 2.2\ 1.3)$
10 $(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$
11 $(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$
12 $(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2\ 2.3})$
13 $(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$
14 $(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$
15 $(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$,

whose number corresponds to the 15 trito-numbers of the polycontextural contexture T_4 (cf. Kronthaler 1986, p. 34), which underlines the fact that these 15 pre-semiotic sign classes are both quantitative and qualitative sign classes, because the integration of the zeroness into the triadic sign relation bridges the polycontextural border between the ontological space of objects and the semiotic space of signs (cf. Bense 1975, p. 65; Toth 2003).

Moreover, we notice that $SR_{4,3}$, unlike the systems SR_3 and SR_4, does not have a dual-identical sign class. On the other side, $SR_{4,3}$ displays, in the system of its dual reality thematics, semiotic structures that do neither occur in SR_3 nor in SR_4. Finally, in $SR_{4,3}$, we do not get any type of n-atomic n-ads, but the following system of **3 tetradic pentatomies** to which the 15 pre-semiotic sign classes can be ordered:

1 $(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$
2 $(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$
4 $(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$
7 $(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$

5 $(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$

11 $(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$
3 $(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$
6 $(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$
10 $(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$
9 $(3.1\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ 2.2\ 1.3)$

14 $(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$
12 $(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2\ 2.3})$
13 $(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$
14 $(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$
8 $(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$

5. As it was shown in Toth (2008c, d),

$SR_{4,3} \not\subset SR_4$,

since the category of zeroness appears only as tetradic, not as trichotomic semiotic value. Moreover, since zeroness (0.) or qualitiy (Q) localizes SR_3 in the ontological space (Bense 1975, p. 65), we also have

$SR_3 \not\subset SR_{4,3}$,

so that, by transitivity,

$SR_3 \not\subset SR_{4,3} \not\subset SR_4$,

and since we found above that

$SR_2 \not\subset SR_3$,

we finally obtain

$SR_2 \not\subset SR_3 \not\subset SR_{4,3} \not\subset SR_4$,

which means that the dyadic Saussurean sign relation is not a sub-relation of the triadic-trichotomic Peircean sign relation, the Peircean sign relation is not a sub-relation of the tetradic-trichotomic) pre-semiotic sign relation, and the latter is not a sub-relation of the tetradic-tetratomic sign relation, either!

However, it is true, from an exclusively quantitative standpoint, that we can visualize an “inclusion” relation between the four sign relations in the following semiotic matrix:

	.0	.1	.2	.3
0.	0.0	0.1	0.2	0.3
1.	1.0	1.1	1.2	1.3
2.	2.0	2.1	2.2	2.3
3.	3.0	3.1	3.2	3.3,

but in doing so, we ultimately "monocontexturalize" all higher semiotic relations down to the dyadic Saussurean "sign relation", which is not even a sign relation, but a dyadic sub-relation, namely the denomination relation of the complete triadic sign relation. Since the Saussurean sign relation corresponds exactly to the semiotic status of numbers in monocontextural mathematics, the following two systems of monocontexturalization of the four sign relations:

(I) $SR_4 \rightarrow SR_3 \rightarrow SR_2$

(II) $SR_{4,3} \rightarrow SR_3 \rightarrow SR_2$

correspond to the reversal of fiberings from the system of Peano numbers into the system of polycontextural numbers (cf. Kronthaler 1986, pp. 93 s.). However, in semiotics, we have two different levels of semiotic monocontexturalization: In (I), the monocontexturalization goes strictly over the abolishment of categories, in $SR_3 \rightarrow SR_2$, the abolisment of the category of thirdness breaks down the "bridge" between world and consciousness or object and subject and turns the triadic sign relation into an "unsaturated" or "partial" sub-sign relation (Bense 1975, p. 44). Such a "sign relation" is thus beneath the recognition of a polycontextural border between sign and object, and this "sign relation" therefore cannot mediate between them. In (II), the monocontexturalization $SR_{4,3} \rightarrow SR_3$ abolishes the quality of zeroness and thus the qualitative embedding of SR_3; with the loss of this strictly qualitative category, the sign relation cannot mediate anymore between the levels of keno- and morphogrammatics on the one side, and semiotics on the other side, thus the polycontextural border between semiotic and ontological space (Bense 1975, p. 65) is abolished.

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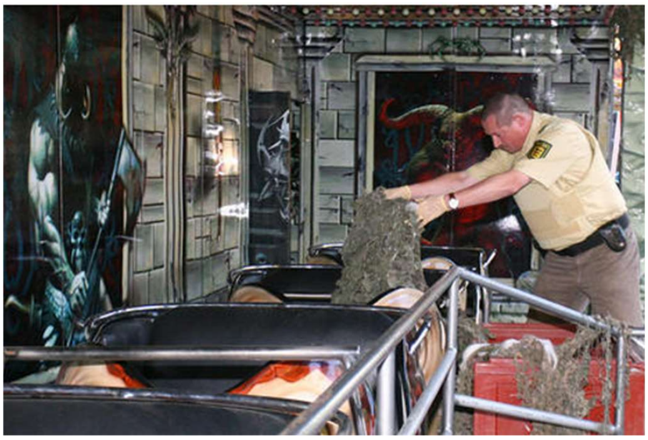
Clockwise and counterclockwise semiotic paths

1. The idea to write this study originated in a discussion with my late friend, the fairground exhibitor Philippe Steiner (cf. Toth et al. 1999; Toth 2000). Philippe owned an old dark ride (also sometimes called ghost train as a calque from German Geisterbahn) through which the cars run counterclockwise, while in most modern dark rides, they run clockwise:



Counterclockwise instead of clockwise orientation is also used in mathematics, e.g., in counting the quadrants of a Cartesian coordinate system, in the labeling of ordered graphs, etc. Moreover, the entrance of most American food stores is to the right, while the exit is to the left for a person who stands in front of the store. Once entered, this person is directed by the architecture of the store to proceed his path through in counterclockwise direction. Would he decide to choose a clockwise path, then he had to navigate himself through the lines of the people standing in front of the cash registers which are situated between the entrance and the exist of the store.

Thus, the question arises if the space concepts of dark rides gave the model for the space concepts in supermarkets or vice versa. As a matter of fact, the stores of the former Swiss chain "Pick Pay" were constructed accurately according to the basic space concept of dark rides: To reach a maximum of length of the path between entrance and exit by as many curves as possible in a pre-given limited space. Needless to say that the paths through the Pick-Pay stores were also counterclockwise. Moreover, in a Pick Pay store, it was impossible to pass by somebody in front of you with the cart, because the paths were corridors hemmed by the shelves. In dark rides, the order of the cars to drive is successive and never simultaneous, too. In Pick Pay stores, it was normally not possible to see somebody passing by in a parallel corridor. In ghost trains, dark screens shield parallel corridors from one another.

Generally, one may say that the smaller the surface of a dark ride is, the easier it can be transported from fairground to fairground, but, at the same time, the more curves have to be constructed in order to reach the maximal time to drive through. The smaller a supermarket is, the curvier its main path has to be in order to displaying a maximal amount of goods. Thus, both in the case of dark rides and in the case of supermarkets, the principle is optimization. Yet, the question stands why newer supermarkets and older dark rides seem to prefer counterclockwise orientation. The often quoted reason, that the dark rides took over their counterclockwise orientation from the older carrousels, in which the direction goes back to the

18th century custom of sticking with a sword into a ring that was fixed on the middle beam of the first carrousels (cf. Dering 1986), is possibly wrong, since then the sticking had to be done left-handed.

2. At the hand of the transpositions of sign classes and reality thematics and their respective cyclic groups (Toth 2008d), in the present study, I will show all possible cycles of transpositions concerning the clockwise or counterclockwise orientation of their semioses. It turns out that counterclockwise orientation appears to be the more “natural” orientation on the level of deepest semiotic representation and thus a sign-theoretic ordering type that is common to all phenomena discussed above, and many more, which are related to the general concept of chirality. This study continues my basic theory of paths (“tracks”) in “Semiotic Ghost Trains” (Toth 2008a) as well as my attempts for a semiotics of time (Toth 2008b, c).

3. First, we introduce the 6 possible order types for each sign class and reality thematics. As a concrete example, we will use the sign class (3.1 2.1 1.3) and its dual reality thematic (3.1 1.2 1.3). Then, we show the different order types using a simple system of arrows and give the respective 2 · 6 possible transpositions also in category theoretic notation:

1. $(3.) \rightarrow (2.) \rightarrow (1.) \times (.1) \leftarrow (.2) \leftarrow (.3)$
$(3.1\ \ 2.1\ \ 1.3) \times (3.1\ \ 1.2\ \ 1.3)$
$[[\beta°, id1], [\alpha°, \beta\alpha]] \times [[\alpha°\beta°, \alpha], [id1, \beta]]$
Order type: rightward

2. $(3.) \rightarrow (1.) \rightarrow (2.) \times (.2) \leftarrow (.1) \leftarrow (.3)$
$(3.1\ \ 1.3\ \ 2.1) \times (1.2\ \ 3.1\ \ 1.3)$
$[[\alpha°\beta°, \beta\alpha], [\alpha, \alpha°\beta°]] \times [[\beta\alpha, \alpha°], [\alpha°\beta°, \beta\alpha]]$
Order type: clockwise

3. $(2.) \rightarrow (1.) \rightarrow (3.) \times (.3) \leftarrow (.1) \leftarrow (.2)$
$(2.1\ \ 1.3\ \ 3.1) \times (1.3\ \ 3.1\ \ 1.2)$
$[[\alpha°, \beta\alpha], [\beta\alpha, \alpha°\beta°]] \times [[\beta\alpha, \alpha°\beta°, \alpha°\beta°, \alpha]]$
Order type: clockwise

4. $(1.) \rightarrow (2.) \rightarrow (3.) \times (.3) \leftarrow (.2) \leftarrow (.1)$
$(1.3\ \ 2.1\ \ 3.1) \times (1.3\ \ 1.2\ \ 3.1)$
$[[\alpha, \alpha°\beta°], [\beta, id1]] \times [[id1, \beta°], [\beta\alpha, \alpha°]]$
Order type: leftward

5. $(1.) \rightarrow (3.) \rightarrow (2.) \times (.2) \leftarrow (.3) \leftarrow (.1)$
$(1.3\ \ 3.1\ \ 2.1) \times (1.2\ \ 1.3\ \ 3.1)$
$[[\beta\alpha, \alpha°\beta°], [\beta°, id1]] \times [[id1, \beta], [\beta\alpha, \alpha°\beta°]]$
Order type: counterclockwise

6. $(2.) \rightarrow (3.) \rightarrow (1.) \times (.1) \leftarrow (.3) \leftarrow (.2)$
$(2.1\ \ 3.1\ \ 1.3) \times (3.1\ \ 1.3\ \ 1.2)$
$[[\beta, id1], [\alpha°\beta°, \beta\alpha]] \times [[\alpha°\beta°, \beta\alpha], [id1, \beta°]]$
Order type: counterclockwise

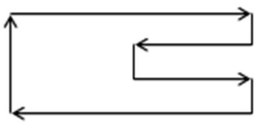
All possible cases of finite and infinite semiotic cycles can be ordered in 3 groups:

1st semiotic cycle:

1. (3.1 2.1 1.3) → (1.3 2.1 3.1) → (3.1 2.1 1.3)

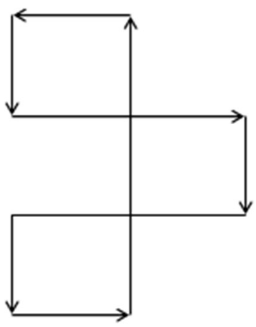
This finite order type is strictly clockwise.

2. (3.1 1.3 2.1) → (2.1 1.3 3.1) → (3.1 1.3 2.1) → ∞



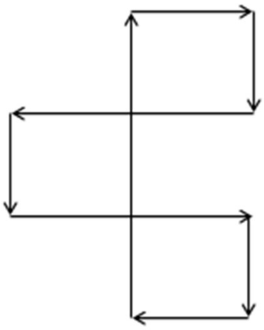
This infinite order type is clockwise, but with one counterclockwise detour.

3. (2.1 3.1 1.3) → (1.3 3.1 2.1) → (2.1 3.1 1.3) → ∞



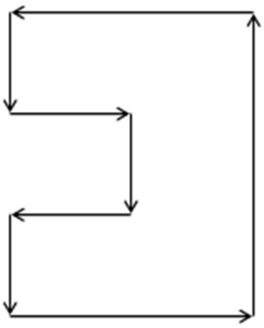
This infinite order type is basically counterclockwise, but with two clockwise detours.

4. (2.1 1.3 3.1) → (3.1 1.3 2.1) → (2.1 1.3 3.1) → ∞



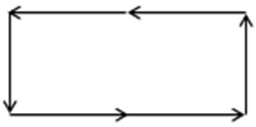
This infinite order type is basically clockwise, but with two counterclockwise detours.

5. (1.3 3.1 2.1) → (2.1 3.1 1.3) → (1.3 3.1 2.1) → ∞



This infinite order type is basically counterclockwise, but with two clockwise detours.

6. (1.3 2.1 3.1) → (3.1 2.1 1.3) → (1.3 2.1 3.1) → ∞



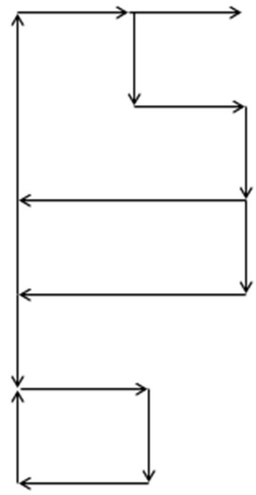
This infinite order type is strictly counterclockwise.

2nd semiotic cycle:

1. (3.1 2.1 1.3) → (2.1 1.3 3.1) → (1.3 3.1 2.1) → (3.1 2.1 1.3)

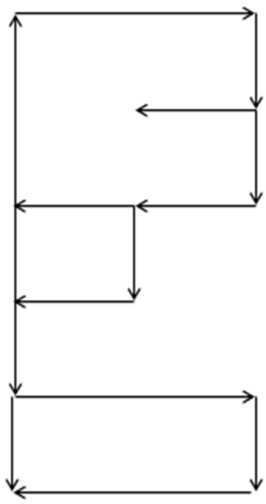
2nd semiotic cycle:

1. (3.1 2.1 1.3) → (2.1 1.3 3.1) → (1.3 3.1 2.1) → (3.1 2.1 1.3)



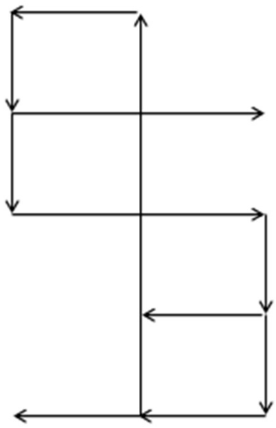
This finite order type is basically clockwise, but with three counterclockwise detours.

2. (3.1 1.3 2.1) → (1.3 2.1 3.1) → (2.1 3.1 1.3) → (3.1 1.3 2.1) → ∞



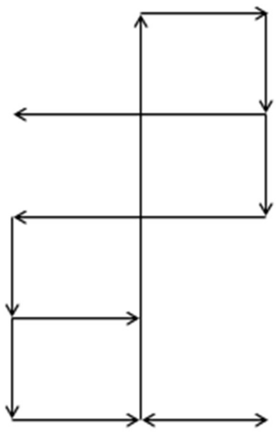
This infinite order type is basically clockwise, but with four counterclockwise detours.

3. (2.1 3.1 1.3) → (3.1 1.3 2.1) → (1.3 2.1 3.1) → (2.1 3.1 1.3) → ∞



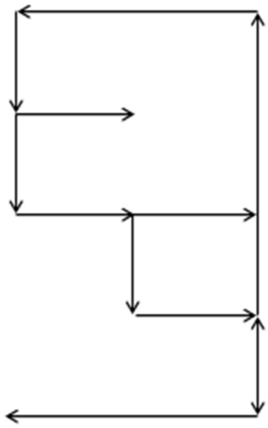
This infinite order type is basically counterclockwise, but with two clockwise detours.

4. (2.1 1.3 3.1) → (1.3 3.1 2.1) → (3.1 2.1 1.3) → (2.1 1.3 3.1)



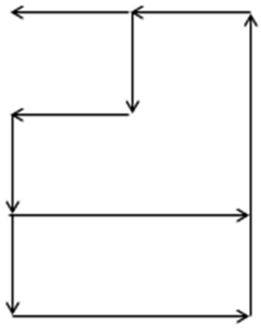
This finite order type is basically clockwise, but with three counterclockwise detours.

5. (1.3 3.1 2.1) → (3.1 2.1 1.3) → (2.1 1.3 3.1) → (1.3 3.1 2.1)



This finite order type is basically counterclockwise, but with three clockwise detours.

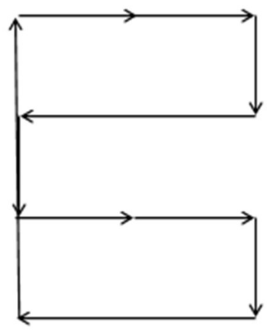
6. (1.3 2.1 3.1) → (2.1 3.1 1.3) → (3.1 1.3 2.1) → (1.3 2.1 3.1) → ∞



This infinite order type is basically counterclockwise, but with two clockwise detours.

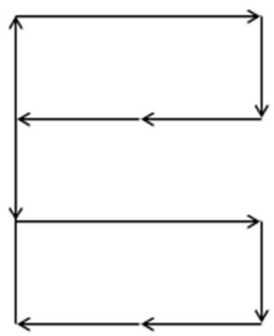
3rd semiotic cycle:

1. (3.1 2.1 1.3) → (1.3 3.1 2.1) → (2.1 1.3 3.1) → (3.1 2.1 1.3)



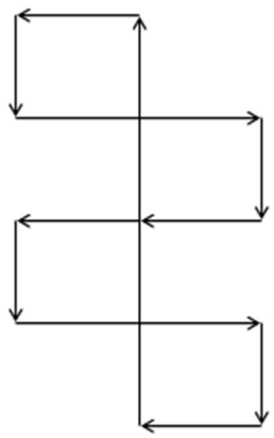
This finite order type is basically clockwise, but with two counterclockwise detours.

2. (3.1 1.3 2.1) → (2.1 3.1 1.3) → (1.3 2.1 3.1) → (3.1 1.3 2.1) → ∞



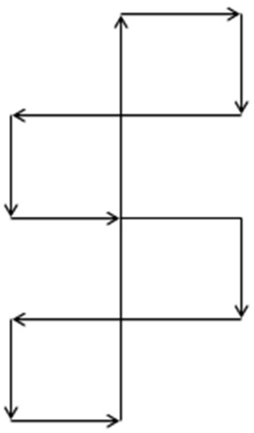
This infinite order type is basically clockwise, but with two counterclockwise detours.

3. (2.1 3.1 1.3) → (1.3 2.1 3.1) → (3.1 1.3 2.1) → (2.1 3.1 1.3) → ∞



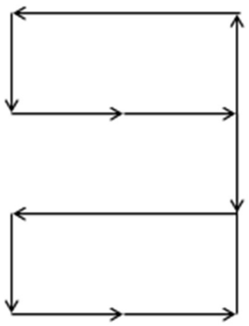
This infinite order type is basically counterclockwise, but with two clockwise detours.

4. (2.1 1.3 3.1) → (3.1 2.1 1.3) → (1.3 3.1 2.1) → (2.1 1.3 3.1)



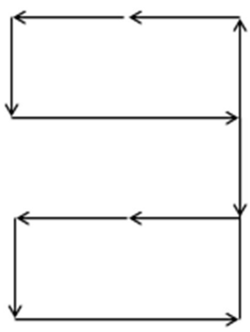
This finite order type is basically clockwise, but with two counterclockwise detours.

5. (1.3 3.1 2.1) → (2.1 1.3 3.1) → (3.1 2.1 1.3) → (1.3 3.1 2.1)



This finite order type is basically counterclockwise, but with two clockwise detours.

6. (1.3 2.1 3.1) → (3.1 1.3 2.1) → (2.1 3.1 1.3) → (1.3 2.1 3.1) → ∞



This infinite order type is basically counterclockwise, but with two clockwise detours.

4. As we recognize, paths that are oriented counterclockwise, are slightly in the overweight over paths that are oriented clockwise insofar as the leftward semioses are concerned. The above 3 semiotic cycles and their 6 order types each show all basic types of semiotic cyclic groups with finite and infinite cycles, whereby the orientation of the paths is uniformly distributed over the 3 semiotic cycles. Thus, the difference between leftward and rightward orientation, parallel and antiparallel structures, chirality, and related structures are already present on the deepest representation level of semiotics.[1] This study therefore confirms the results of Ertekin Arin from architecture semiotics, especially about "adaptation iconism" (Arin 1981, pp. 280 ss.; Arin 1984).

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[1] Nevertheless, the priority of right before left, up before down, etc. seems to be a culturally determined phenomenon, as, e.g. ungrammatical English "binomials" like "left and right", "down and up", "fro and to", etc. show – quite opposite, e.g. to Hawaiian and other Polynesian languages (cf. Elbert and Pukui 1979; Toth 2008e).

On the genesis of semiosis

1. Bense (1967, p. 9) writes laconically: “Sign is everything, that is introduced as a sign, and only what is introduced as a sign. Each arbitrary thing can (principally) be introduced as a sign. What has been introduced as a sign, is no longer an object anymore, but an assignment (to a thing that can be object); so to speak a meta-object”. More explicitly, we read in Bense and Walther (1973, p. 26): “Introduction of a sign means that a sign is not given like an object of nature, but is introduced by a consciousness. This introduction can be understood as ‘setting’, ‘declaration’ and thus as ‘selection’. Therefore, a sign can only be understood as a ‘thetic’ item, it has a principal ‘thetic’ character”.

2. The introduction of a sign for an object allows using this object and referring to it independently from its local and temporal position and thus “frees” it from its geographical boundaries. However, apparently, there are three kinds of representations of an object by a sign:

2.1. If an object itself is taken for a sign, then sign and object contain one another, either as part or proper part; moreover, they are necessarily similar to one another. This is, what Peirce calls the iconic object-relation of a sign (2.1). Thus, an icon has the shortest local and temporal distance to its object.

2.2. If a sign refers to a distant object, like a signpost indicates the direction of a town that is locally and temporally absent from it, then sign and object do not stand in a relation of parthood, but in a nexal relation. Peirce calls this the indexical object-relation of a sign (2.2). Pure indices are not similar to their objects. In pictograms, their icons are redundant from the viewpoint of the indexical function, but this redundancy is intended to reduce the entropy of the index, which naturally results from its nexal, non-parthood relationship.

2.3. Even farther away from the object it is referring to, is, what Peirce calls a symbol (2.3). Only the symbolic sign is completely disjoint and thus free from the object it refers to. Therefore, a pure symbol has no similarity with its object. The similarity of onomatopoeic words is due to the iconic character of these symbols, which is also redundant, but is intended to reduce the entropy of the symbol, which naturally results from its complete independence from its object.

3. Looking at the three object-relations of a sign in this way, it is obvious that in the progress between icon (2.1), index (2.2) and symbol (2.3), the maximal evidence of the referred object in (2.1), which gets fragile in (2.2), vanishes in (2.3) (cf. Toth 2008, pp. 286 ss.). This presupposes that the iconic object-relation of a sign is older, from the standpoint of phylogenetics, and that the progress (2.1) > (2.2) > (2.3) does not only represent the increasing freedom of a sign from its objects, but also the entropy of reference of this sign to its objects. Thus, semiotic redundancy also increases from the icon (2.1) to the index (2.2) and to the symbol (2.3). At the same time, indices are redundantly used together with symbols in order to establish a nexal framework for completely arbitrary signs, and indices are redundantly used together with icons in order to specify the local and temporal settings of the object referred to by the icon. These strategies of redundancy serve to diminish the entropy inherent in object-relations of signs that inherited this entropy by the process of their liberation from their referred objects. Redundancy can thus be interpreted as a counter-movement against the decreasing evidence, which results from increasing freedom of a sign in respect to its object.

4. Therefore, in a triadic sign-relation, that contains the monadic relation of the medium or sign-carrier (.1.), the dyadic relation of the referred object (.2.), and the triadic relation of the consciousness of interpretation (.3.), the part-relation between the medium and the object are basic. In the case of iconic representation, the medium is nothing else than the object, after it has been declared as a sign by the consciousness, and thus, what Bense calls a meta-object.

4.1. The icon represents its object by the following semiotic connection:

$(2.1) \times (1.2)$,

This means, that an object that is declared as a sign, can only use a singular sign-carrier for its representation. This is obvious, since the icon stands in a parthood-relationship to its referred object, and a parthood-relationship is defined through the relation between an element and the set to which this element belongs.

4.2. Since the dyadic relation of designation (.1. $\Rightarrow$ .2.) between an iconic object and its substituting singular medium is thus (2.1 1.2), it follows that the most fundamental sign class to represent any objects is

(3.1 2.1 1.2),

together with the most fundamental reality thematic that stands to the sign-class in the relation of dualization

(2.1 1.2 1.3).

Thus, the most fundamental structural reality presented by a reality thematic of a sign class is

(2.1)-thematized (1.2 1.3), i.e. a medium-thematized object,

or an iconic object (2.1) represented by either a singular (1.2) or an arbitrary (1.3) medium (sign carrier). The singular medium refers to the case where the sign is a part of its object (pars pro toto relation); the arbitrary medium refers to the case where the sign is not contained by its object. Therefore, the maximally open consciousness, the rhematic interpretant (3.1), creates the arbitrary medium

(3.1×1.3),

and the arbitrary medium creates the maximally open interpretant relation

(1.3×3.1).

If signs are not represented through arbitrary sign carriers, their dual reality thematics cannot establish open interpretative connexes and thus a triadic relation over the dyadic designation relation between sign and object, and vice versa. A sign that can only be represented by a singular medium, establishes, via dualization, only the object-relation of its sign relation and thus remains dyadic.

4.3. Again in other words, the most basic semiotic dualization

(2.1×1.2)

marks the primordial **semiotic difference** between a sign and its object. At the same time, this relation of dualization sets the two semiotic relations, the dyadic iconic object-relation (2.1) and the monadic singular medium (1.2), in semiotic **opposition** to one another. Therefore, difference and opposition as sources of semiosis do not only appear after a full triadic sign relation is established (as was assumed, amongst others, by de Saussure (1916) and Nöth (1994)), but they are **pre-existent** to the act of thetic introduction of a sign or transformation of an object into a meta-object. Furthermore, as one recognizes, **difference is primordial to opposition**, hence opposition establishes only after a difference has been made (cf. Spencer Brown 1969).

4.4. However, the triadic interpretant relation, which is connected over the dyadic relation of designation $(.1. \Rightarrow .2.)$, implies a third semiotic value, after the value for the object (.2.) and the value for the medium (.1.) have been introduced. However, this third semiotic value cannot be taken from the basic dyadic relation (2.1×1.2) of semiotic difference, and thus, in a mono-contextural world of binary logic, must be taken from the semiotic identity relation

(1.1 2.2 3.3),

which has been called by Bense the "Genuine Category Class" (Bense 1992, pp. 27 ss.). Therefore, **semiotic identity is posterior to semiotic difference**.

As soon as the semiotic identity relation is established, all other (32 – 2) = 7 sub-signs can be constructed, which is shown best by using the semiotic matrix, in which the 9 sub-signs appear as Cartesian products of the mapping of the triadic sign-relation (.1., .2., .3.) into itself

$(.1., .2., .3.) \times (.1., .2., .3.) =$

$$\begin{bmatrix} 1.1 & 1.2 & 1.3 \\ 2.1 & 2.2 & 2.3 \\ 3.1 & 3.2 & 3.3 \end{bmatrix}$$

Therefore, most basically, it is enough to have the basic semiotic object-relation

$(2.1) \times (1.2)$,

the operation of dualization

$\times := (a.b) \rightarrow (b.a)$,

and the Genuine Category Class, which consists of the identitive morphisms idx:

(1.1 2.2 3.3).

On the basis of these two relations and one operation, all sub-signs can be created, and all other semiotic relations of the sign-relation (.3., .2., .1.) can be constructed.

4.5. Since the 9 sub-signs from the semiotic matrix are restricted to appear in a triadic sign relation (3.a 2.b 1.c) by the semiotic inclusion order

a ≤ b ≤ c,

the total amount of sign classes is not 3 · 3 · 3 = 27, but only 10 sign classes, which we will order here according to their object-relations, and which allows us to group them into the following three classes of 3 sign-classes with iconic (2.1), 4 sign-classes with indexicalic (2.2), and 3 sign-classes with symbolic (2.3) object-relation:

3.1 2.1 .... 1.1
3.1 2.1 .... 1.2
3.1 2.1 .... 1.3

3.1 2.2 .... 1.2
3.1 2.2 .... 1.3
3.2 2.2 .... 1.2
3.2 2.2 .... 1.3

3.1 2.3 .... 1.3
3.2 2.3 .... 1.3
3.3 2.3 .... 1.3

As we recognize, the sign classes with iconic (2.1) object-relation are connected, via dualization, with their medium or sign carrier:

$$\begin{array}{c|c|c c c|c|c}
\hline
3.1 & 2.1 & 1.1 & _\times & 1.1 & 1.2 & 1.3 \\
3.1 & 2.1 & 1.2 & _\times & 2.1 & 1.2 & 1.3 \\
3.1 & 2.1 & 1.3 & _\times & 3.1 & 1.2 & 1.3 \\
\hline
\end{array}$$

The sign classes with indexicalic (2.2) object-relation are self-connected:

$$\begin{array}{c|c|c c c|c|c}
\hline
3.1 & 2.2 & 1.2 & _\times & 2.1 & 2.2 & 1.3 \\
3.1 & 2.3 & 1.3 & _\times & 3.1 & 2.2 & 1.3 \\
3.2 & 2.2 & 1.2 & _\times & 2.1 & 2.2 & 2.3 \\
3.2 & 2.2 & 1.3 & _\times & 3.1 & 2.2 & 2.3 \\
\hline
\end{array}$$

And the sign-classes with symbolic object-relation (2.3) are connected, via dualization, with their interpretant relation:

$$\begin{array}{c|c|c c c|c|c}
3.1 & 2.3 & 1.3 & _\ \times & 3.1 & 3.2 & 1.3 \\
3.2 & 2.3 & 1.3 & _\ \times & 3.1 & 3.2 & 2.3 \\
3.3 & 2.3 & 1.3 & _\ \times & 3.1 & 3.2 & 3.3
\end{array}$$

In other words: A sign with iconic (2.1) object-relation does not automatically establish an interpretative connex over its dyadic designation relation (2.1 × 1.2), while a sign with symbolic (2.3) object-relation does (2.3 × 3.2). The signs with indexicalic (2.2) object-relation appear as mediative sign classes in which the signs refer to their objects by referring to themselves, since the index appears also in their dual reality thematics as index.

4.6. Besides the fundamental semiotic difference relation (2.1 × 1.2), there is only one more basic difference relation:

(3.1 × 1.3),

since all other dual sign-relations are not basic. This second semiotic difference relation appears only in one of the self-referential sign classes with indexicalic object-relation:

(3.1 2.2 1.3)

and is both dual-invariant

(3.1 2.2 1.3) × (3.1 2.2 1.3)

and symmetric

(3.1 2×2 1.3).

The dual-invariance of the sign-class (3.1 2.2 1.3) says that there is no semiotic difference between the sign and its represented reality. The symmetric structure of both sign class and reality thematic shows that the self-referential indexicalic object relation (2.2) is embedded into the basic dual sign relation (1.3 × 3.1). Therefore, the sign class (3.1 2.2 1.3) was considered by Max Bense (1992) the sign class of the sign itself, i.e. this sign relation represents the sign itself, whose dual reality thematic is identical with the sign class. Moreover, Walther (1982) showed that all other 9 sign classes and 9 reality thematics are connected by at least one and maximally two sub-signs with this sign class, which Bense called "eigenreal". Therefore, the dual-identical eigenreal sign class is the only sign class, constructed over the sign-relation $SR_{3,3}$, which combines a basic semiotic difference relation (1.3 × 1.3) with an identitive morphism (2.2). Hence, in the sign class (3.1 2.2 1.3), semiotic difference and semiotic identity are combined. However, nevertheless, the origin of semiosis starts with the sign class (3.1 2.1 1.2), that represents, according to Bense (1983, pp. 53 s.) "natural" signs like "rests" or "traces", that are "parts of an object". Thus, the sign, and with it semiosis, starts, as has been assumed up to now, with natural signs, and as semiotic identity is posterior to semiotic difference, "artificial" signs, and amongst them the relation of a sign to itself in its eigenreality, are posterior to "natural" signs, whose phylogenetic ancienneté has also been shown by various authors.

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Intra-semiotic connections and structural realities of pre-semiotics

1. In Toth (2008b, pp. 28 ss.), I have introduced the differentiation between intra- and extra-semiotic connections, based on the system of the 10 sign classes and their 10 dual reality thematics over the triadic-trichotomic sign relation $SR_{3,3}$. In the present study, I will show the intra-semiotic connections of the 15 sign classes and their reality thematics over the tetradic-trichotomic pre-semiotic sign relation $SR_{4,3}$ (cf. Toth 2008c, d, e) and investigate the pre-semiotic structural reality thematics, which are presented by the pre-semiotic reality thematics.

2. In the following list, I display the intra-semiotic connections of the 15 pre-semiotic sign classes and their dual reality thematics together with their ports or sets of shared sub-signs:

16 (3.1 2.1 $\underline{1.1}$ 0.1) × (1.0 $\underline{1.1}$ 1.2 1.3) {<(1.1)>}
17 (3.1 2.1 $\underline{1.1}$ 0.2) × (2.0 $\underline{1.1}$ 1.2 1.3) {<(1.1)>}
18 (3.1 2.1 $\underline{1.1}$ 0.3) × (3.0 $\underline{1.1}$ 1.2 1.3) {<(1.1)>}
19 (3.1 $\underline{2.1\ 1.2}$ 0.2) × (2.0 $\underline{2.1\ 1.2}$ 1.3) {<(2.1, 1.2)>}
20 (3.1 $\underline{2.1\ 1.2}$ 0.3) × (3.0 $\underline{2.1\ 1.2}$ 1.3) {<(2.1, 1.2)>}
21 ($\underline{3.1}$ 2.1 $\underline{1.3}$ 0.3) × (3.0 $\underline{3.1}$ 1.2 $\underline{1.3}$) {<(3.1, 1.3)>}
22 (3.1 $\underline{2.2}$ 1.2 0.2) × (2.0 2.1 $\underline{2.2}$ 1.3) {<(2.2)>}
23 (3.1 $\underline{2.2}$ 1.2 0.3) × (3.0 2.1 $\underline{2.2}$ 1.3) {<(2.2)>}
24 ($\underline{3.1\ 2.2\ 1.3}$ 0.3) × (3.0 $\underline{3.1\ 2.2\ 1.3}$) {<(3.1), (2.2), (1.3)>}
25 ($\underline{3.1}$ 2.3 $\underline{1.3}$ 0.3) × (3.0 $\underline{3.1}$ 3.2 $\underline{1.3}$) {<(3.1), (1.3)>}
26 (3.2 $\underline{2.2}$ 1.2 0.2) × (2.0 2.1 $\underline{2.2}$ 2.3) {<(2.2)>}
27 (3.2 $\underline{2.2}$ 1.2 0.3) × (3.0 2.1 $\underline{2.2}$ 2.3) {<(2.2)>}
28 (3.2 $\underline{2.2}$ 1.3 0.3) × (3.0 3.1 $\underline{2.2}$ 2.3) {<(2.2)>}
29 ($\underline{3.2\ 2.3}$ 1.3 0.3) × (3.0 3.1 $\underline{3.2\ 2.3}$) {<(3.2), (2.3)>}
30 ($\underline{3.3}$ 2.3 1.3 0.3) × (3.0 3.1 3.2 $\underline{3.3}$) {<(3.3)>}

Although SS15 does not contain any dual-identical sign classes, no. 9 shows triadic semiotic connection between the sign class and its reality thematic.

2. In the following, we will now consider the structural realities of the pre-semiotic reality thematics and compare them to the ports. The abbreviation Thn stands for "thematizing relation of sub-signs":

1.	(1.0 $\underline{1.1\ 1.2\ 1.3}$)	M-them. M	P = {<(1.1)>}	$P \subset Thn$
2.	(2.0 $\underline{1.1\ 1.2\ 1.3}$)	M-them. O	P = {<(1.1)>}	$P \subset Thn$
3.	(3.0 $\underline{1.1\ 1.2\ 1.3}$)	M-them. I	P = {<(1.1)>}	$P \subset Thn$
4.	($\underline{2.0\ 2.1\ 1.2}$ 1.3)	O-them. M	P = {<(2.1, 1.2)>}	$P \subset Thn$
5.	(3.0 2.1 $\underline{1.2\ 1.3}$)	M-them. I/O	P = {<(2.1, 1.2)>}	$P_1 \subset Thn$, $P_2 \subset Thn$

6. $(\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$	M-them. I	$P = \{<(3.1, 1.3)>\}$	$P_1 \subset Thn, P_2 \subset Thn$
7. $(\underline{2.0\ 2.1\ 2.2}\ 1.3)$	O-them. M	$P = \{<(2.2)>\}$	$P \subset Thn$
8. $(3.0\ \underline{2.1\ 2.2}\ 1.3)$	O-them. I/M	$P = \{<(2.2)>\}$	$P \subset Thn$
9. $(\underline{3.0\ 3.1}\ 2.2\ 1.3)$	I-them. O/M	$P = \{<(3.1), (2.2), (1.3)>\}$	$P_1 \subset Thn, P_2 \subset Thn$
10. $(\underline{3.0\ 3.1\ 3.2}\ 1.3)$	I-them. M	$P = \{<(3.1), (1.3)>\}$	$P_1 \subset Thn, P_2 \subset Thn$
11. $(2.0\ \underline{2.1\ 2.2\ 2.3})$	O-them. O	$P = \{<(2.2)>\}$	$P \subset Thn$
12. $(3.0\ \underline{2.1\ 2.2\ 2.3})$	O-them. I	$P = \{<(2.2)>\}$	$P \subset Thn$
13. $(3.0\ 3.1\ \underline{2.2\ 2.3})$	O-them. I	$P = \{<(2.2)>\}$	$P \subset Thn$
14. $(\underline{3.0\ 3.1\ 3.2}\ 2.3)$	I-them. O	$P = \{<(3.2), (2.3)>\}$	$P_1 \subset Thn, P_2 \subset Thn$
15. $(3.0\ \underline{3.1\ 3.2\ 3.3})$	I-them. I	$P = \{<(3.3)>\}$	$P \subset Thn$

Thus, in the pre-semiotic structural realities presented by the reality thematics of nos. 5,6, 9, 10, and 14, the sets of ports which consist of 2 or 3 sub-sets, are distributed over the part-relations of thematizing and thematized realities. If we compare no. 4 and no. 5, we recognize that this relational distribution is independent of the number of sub-sets of the pre-semiotic port-sets, yet, of course, a port with only one element cannot be distributed over two part-relations. Moreover, the only case with triadic intra-semiotic connection (no. 9) displays only two port-sub-sets.

3. In Toth (2008a, pp. 177 ss.), I have shown that each triadic-trichotomic sign class has 3! = 6 permutations. Therefore, each tetradic-trichotomic sign class has 4! = 24 permutations and thus 24 pre-semiotic sign classes, reality thematics and structural realities. In the following, we will show the great impact of permutations to the differentiation of structural realities especially in pre-semiotics. As an example, we will take the pre-semiotic dual system $(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ 1.2\ 1.3)$ with its port $\{<(2.1, 1.2)>\}$ and $P_1 \subset Thn$, $P_2 \subset Thn$. As we will see, permutations turn out to be a major tool in distributing sets of intra-semiotic connections over part-relations of thematizing and thematized realities. For the notation of the structural realities, cf. Toth (2008a, pp. 223 ss.). As above, sets of thematizing sub-signs are underlined; the port-relations are bold.

$(\underline{1.3\ \mathbf{1.2}}\ \mathbf{2.1}\ 3.0)$	$(1^{2,>} \rightarrow 2^1 \leftrightarrow 3^1)$	$(\mathbf{2.1}\ 3.0\ \underline{\mathbf{1.2}\ 1.3})$	$(2^1 \leftrightarrow 3^1 \leftarrow 1^{2,<})$
$(\underline{\mathbf{1.2}\ 1.3}\ \mathbf{2.1}\ 3.0)$	$(1^{2,<} \rightarrow 2^1 \leftrightarrow 3^1)$	$(\underline{1.3}\ 3.0\ \underline{\mathbf{1.2}}\ \mathbf{2.1})$	$(1^{1,>} \rightarrow 3^1 \leftarrow 1^1 \rightarrow 2^1)$
$(\underline{1.3}\ \mathbf{2.1}\ \underline{\mathbf{1.2}}\ 3.0)$	$(1^{1,>} \rightarrow 2^1 \leftarrow 1^1 \rightarrow 3^1)$	$(\underline{\mathbf{1.2}}\ 3.0\ \underline{1.3}\ \mathbf{2.1})$	$(1^{1,<} \rightarrow 3^1 \leftarrow 1^1 \rightarrow 2^1)$
$(\mathbf{2.1}\ \underline{1.3\ \mathbf{1.2}}\ 3.0)$	$(2^1 \leftarrow 1^{2,>} \rightarrow 3^1)$	$(\mathbf{2.1}\ 3.0\ \underline{1.3\ \mathbf{1.2}})$	$(2^1 \leftrightarrow 3^1 \leftarrow 1^{2,>})$
$(\underline{\mathbf{1.2}}\ \mathbf{2.1}\ \underline{1.3}\ 3.0)$	$(1^{1,<} \rightarrow 2^1 \leftarrow 1^1 \rightarrow 3^1)$	$(\underline{1.3}\ 3.0\ \mathbf{2.1}\ \underline{\mathbf{1.2}})$	$(1^{1,>} \rightarrow 3^1 \leftrightarrow 2^1 \leftarrow 1^1)$
$(\mathbf{2.1}\ \underline{\mathbf{1.2}\ 1.3}\ 3.0)$	$(2^1 \leftarrow 1^{2,<} \rightarrow 3^1)$	$(\underline{\mathbf{1.2}}\ 3.0\ \mathbf{2.1}\ \underline{1.3})$	$(1^{1,<} \rightarrow 3^1 \leftrightarrow 2^1 \leftarrow 1^1)$

$(\underline{1.3\ \mathbf{1.2}}\ 3.0\ \mathbf{2.1})$	$(1^{2,>} \rightarrow 3^1 \leftrightarrow 2^1)$	$(3.0\ \mathbf{2.1}\ \underline{\mathbf{1.2}\ 1.3})$	$(3^1 \leftrightarrow 2^1 \leftarrow 1^{2,<})$
$(\underline{\mathbf{1.2}\ 1.3}\ 3.0\ \mathbf{2.1})$	$(1^{2,<} \rightarrow 3^1 \leftrightarrow 2^1)$	$(3.0\ \underline{\mathbf{1.2}\ 1.3}\ \mathbf{2.1})$	$(3^1 \leftarrow 1^{2,<} \rightarrow 2^1)$
$(\underline{1.3}\ \mathbf{2.1}\ 3.0\ \underline{\mathbf{1.2}})$	$(1^{1,>} \rightarrow 2^1 \leftrightarrow 3^1 \leftarrow 1^1)$	$(3.0\ \underline{1.3}\ \mathbf{2.1}\ \underline{\mathbf{1.2}})$	$(3^1 \leftarrow 1^{1,>} \rightarrow 2^1 \leftarrow 1^1)$
$(\mathbf{2.1}\ \underline{1.3}\ 3.0\ \underline{\mathbf{1.2}})$	$(2^1 \leftarrow 1^{1,>} \rightarrow 3^1 \leftarrow 1^1)$	$(3.0\ \mathbf{2.1}\ \underline{1.3\ \mathbf{1.2}})$	$(3^1 \leftrightarrow 2^1 \leftarrow 1^{2,>})$
$(\underline{\mathbf{1.2}}\ \mathbf{2.1}\ 3.0\ \underline{1.3})$	$(1^{1,<} \rightarrow 2^1 \leftrightarrow 3^1 \leftarrow 1^1)$	$(3.0\ \underline{\mathbf{1.2}}\ \mathbf{2.1}\ \underline{1.3})$	$(3^1 \leftarrow 1^{1,<} \rightarrow 2^1 \leftarrow 1^1)$
$(\mathbf{2.1}\ \underline{\mathbf{1.2}}\ 3.0\ \underline{1.3})$	$(2^1 \leftarrow 1^{1,<} \rightarrow 3^1 \leftarrow 1^1)$	$(3.0\ \underline{1.3\ \mathbf{1.2}}\ \mathbf{2.1})$	$(3^1 \leftarrow 1^{2,>} \rightarrow 2^1)$

Since the structural realities are of course shared by all sign classes, the 24 transpositions of each sign class of SS15 over $SS_{4,3}$ can thus be summed up into 6 types, each of which has one variant, concerning the semiosic order inside of a set of thematzing sub-signs (e.g., (1.2, 1.3) vs. (1.3, 1.2); the semiosic order is thus determined by the trichotomic values of the thematizing sub-signs):

1.a) (12,> → 21 ↔ 31)
1.b) (12,< → 21 ↔ 31)
2.a) (31 ↔ 21 ← 12,>)

2.b) (31 ↔ 21 ← 12,<)

3.a) 31 ← 12,> → 21)
3.b) (31 ← 12,< → 21)

4.a) (11,> → 31 ← 11 → 21)
4.b) (11,< → 31 ← 11 → 21)
5.a) (31 ← 11,> → 21 ← 11)

5.b) (31 ← 11,< → 21 ← 11)

6.a) (11,> → 31 ↔ 21 ← 11)
6.b) (11,> → 31 ↔ 21 ← 11)

As for the sets of ports, the variants do not count for triadic structural realities, since in our notation, the sub-signs with “frequency” 2 are amalgating two sub-signs with identical triadic value. However, in tetradic structural realities, where there can be no amalgamations in the notation of the four sub-signs of a tetradic sign relation, sub-signs with identical triadic, but different trichotomic values appear as two instances, and hence the variants do count. Therefore, we can finally show the distribution of the elements of ports by permutation in pre-semiotic structural realities:

1. (**1**2,> → **21** ↔ 31)
2. (31 ↔ **21** ← **1**2,>)

3. 31 ← **12**,> → **2**1)

4.a) (11,> → 31 ← **11** → **21**)
4.b) (**11**,< → 31 ← 11 → **21**)
5.a) (31 ← 11,> → **21** ← **11**)

5.b) (31 ← **11**,< → **21** ← 11)

6.a) (11,> → 31 ↔ **21** ← **11**)
6.b) (**11**,< → 31 ↔ **21** ← 11)

As we recognize, in triadic structural realities, there is no “splitting” like, e.g., *(**1**2,> → 31 ↔ **21**), while in tetradic structural realities, there are. Types like no. 4.b), in which 2 sub-sets of a set of semiotic ports are split over n-2 sub-signs of an n-adic sign relation, could be called “semiotic stranding”.

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Outlines of a general model for a pre-semiotic space

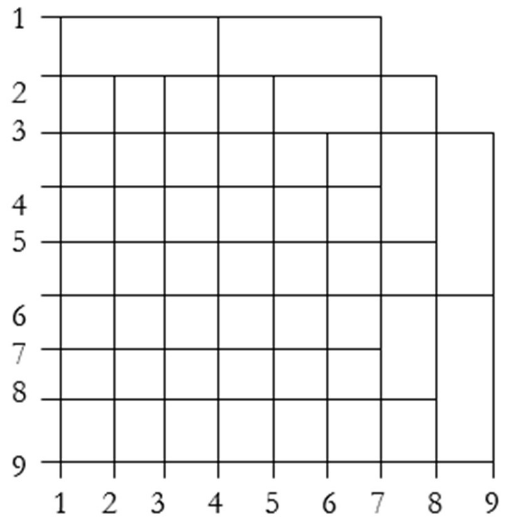
1. My “Semiotic Relational Grammar” (SRG), which had appeared in 1997, was the first attempt at constructing a topological semiotic space by aid of category theory (Toth 1997). SRG is a two-dimensional semiotic space in which only such structural realities are connected to one another, that present the same kind of sets of thematized realities. In the respective graph, the reality thematics that present the structural realities are the vertices and the connections of identical thematizates are the edges.

E.g., in SRG over $SR_{3,3}$, which we will abbreviate as $SRG_{3,3}$, the pair of structural realities to the left does not share the same thematizate, but the two pairs to the right do:

($\underline{3.1\ 3.2}$ 3.3)	($\underline{3.1\ 3.2}$ 2.3)	($\underline{3.1\ 3.2}$ 2.3)
(1.1 $\underline{1.2\ 1.3}$)	(2.1 $\underline{2.2\ 2.3}$)	(2.1 $\underline{1.2\ 1.3}$)

The two structural realities to the left are (I-them. I) and (M-them. M), thus, the thematizates are in the first case an I and in the second case an M, hence they cannot be connected. However, in the two pairs to the right, we have (I-them. O) and (O-them. O) in the first case, and (I-them. O) and (M-them. O) in the second case, hence in both cases a thematized object (although the thematizing sub-signs are not the same), and thus both structural realities of both pairs will be connected.

Since only such structural realities are connected to one another, which show the same thematizates, the graph of SRG has an antimatroidal structure. An antimatroid is a family of sets closed under union, such that for every (nonempty) set in the family there is an element that can be removed to produce another set in the family. The antimatroid-character of SRG is what gives SRG an outer and inner “stairwell-like” appearance:



Antimatroids are also known as “learning spaces”, whose structure can be made apparent by drawings in which all faces are quadrilaterals with the bottom and left sides parallel to the coordinate axis (and where the drawing has unique top-right and bottom-left vertices). “Such a drawing only exists for graphs coming from antimatroids” (Eppstein 2006a). It is even true that “each upright-quad drawing represents an st-planar learning space” (Eppstein 2006b, p. 11).

The nos. 1-9 of the vertices refer to the following structural realities:

1 := ($\underline{3.1\ 3.2}$ 3.3)	I-them. I	6 := ($\underline{2.1\ 2.2}$ 1.3)	O-them. M
2 := ($\underline{3.1\ 3.2}$ 2.3)	I-them. O	7 := (3.1 $\underline{1.2\ 1.3}$)	M-them. I
3 := ($\underline{3.1\ 3.2}$ 1.3)	I-them. M	8 := (2.1 $\underline{1.2\ 1.3}$)	M-them. O
4 := (3.1 $\underline{2.2\ 2.3}$)	O-them. I	9 := ($\underline{1.1\ 1.2}$ 1.3)	M-them. M
5 := ($\underline{2.1\ 2.2}$ 2.3)	O-them. O		

So, if only thematized M, O, I are combined with thematized M, O, I, then $SRG_{3,3}$, as depicted above, has exactly 66 intersects of semiotic relations. For illustration, I show the sign connections of the first leftmost column of $SRG_{3,3}$, i.e. the connections between the subsets for ((1,1), (2,1), (3,1), ..., (9,1)). The left column beneath uses “static” morphisms, the right column “dynamic” morphisms (cf. Toth 2008a, pp. 159 ss., 259 ss.):

(1,1)	[id3, id3, id3]	[[id3, α], [id3, β]]
(2,1)	[id3, id3, β]	[[id3, α], [β°, β]]
(3,1)	[id3, id3, βα]	[[id3, α], [α°β°, β]]
(4,1)	[id3, β, β]	[[β°, α], [id2, β]]
(5,1)	[β, β, β]	[[id2, α], [id2, β]]
(6,1)	[β, β, βα]	[[id2, α], [α°, β]]
(7,1)	[id3, βα, βα]	[[α°β°, α], [id1, β]]
(8,1)	[β, βα, βα]	[[α°, α], [id1, β]]
(9,1)	[βα, βα, βα]	[[id1, α], [id1, β]]

2. As one recognizes, the structure of the connections of $SRG_{3,3}$ is the same from top to bottom and from left to the right, so that the graph is symmetric for rotation. This allows to consider $SRG_{3,3}$ a topologically stratified space. Generally, an n-dimensional topological stratification of a topological space X is a filtration

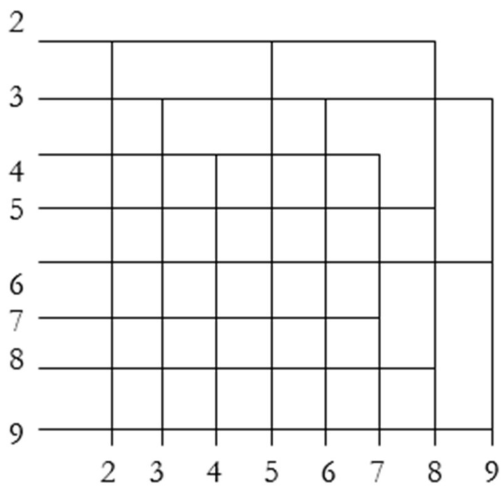
$\varnothing = X_{-1} \subset X_0 \subset X_1 \subset ... \subset X_n = X$

of X by closed subspaces such that for each i and for each point x of $X_i \setminus X_{i-l}$, there exists a neighborhood $U \subset X$ of x in X, a compact n-i-l-dimensional stratified space L, and a filtration-preserving homeomorphism

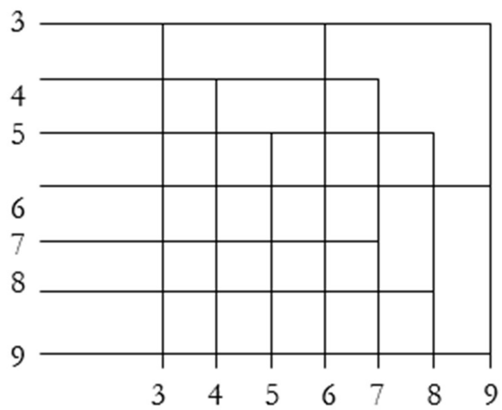
$U \cong \mathbb{R}i \times CL$. Here, CL is the open cone on L. If X is a topologically stratified space, the i-dimensional stratum of X is the space $X_i \setminus X_{i-I}$ (Goresky 1983).

In the case of $SRG_{3,3}$, the stratified spaces are simply the sub-spaces, and there are as many nonempty subspaces as there are nonempty subsets of its carrier set. However, for SRG as a semiotic space, it is senseless to construct 8 subspaces, because then we would get only identical thematizates at the end. Since $SRG_{3,3}$ is constructed from 3 blocks of 3 reality thematics, according to the Trichotomic Triads (cf. Toth 1997, pp. 36 ss.), we obtain the following 6 subspaces, whose last one consists of the self-thematizations of M-them. I, M-them. O, and M-them. M. Therefore, according to the antimatroidal structure of $SRG_{3,3}$, we can construct the following subspaces by letting away step by step one thematization while proceeding downward and rightward from one stratum to the next:

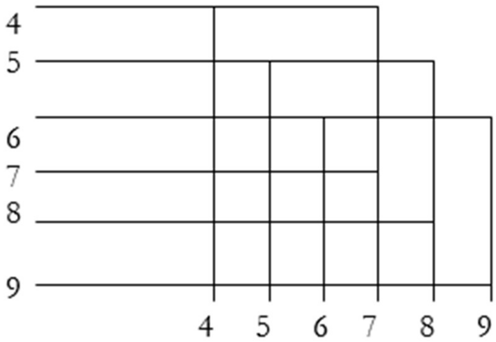
SRG\ (I-I)



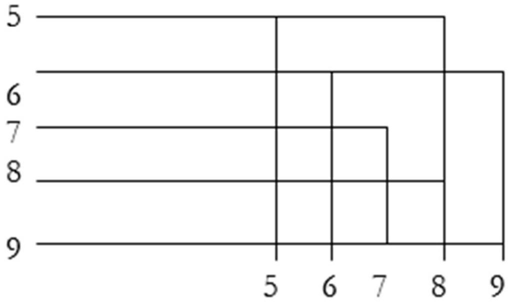
SRG \ (I-I ∧ I-O)



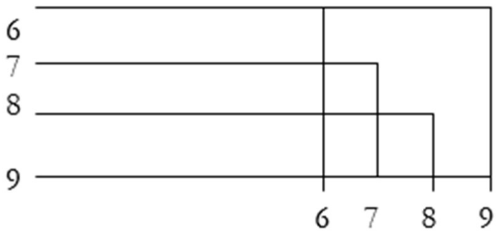
SRG \ (I-I ∧ I-O ∧ I-M)



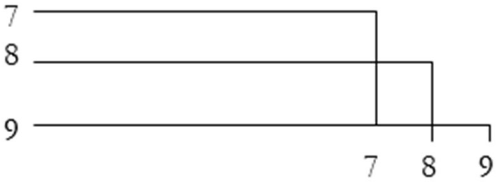
SRG \ (I-I ∧ I-O ∧ I-M ∧ O-I)



SRG \ (I-I ∧ I-O ∧ I-M ∧ O-I ∧ O-O)



SRG \ (I-I ∧ I-O ∧ I-M ∧ O-I ∧ O-O ∧ O-M)



3. We now turn to the set SS15 of 15 sign classes and reality thematics over the pre-semiotic sign relation $SR_{4,3}$:

1	$(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$	M-them. M
2	$(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$	M-O
3	$(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$	M-them. I
4	$(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$	M-them. O / O-them. M (2)
5	$(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$	M-them. O / M-them. I (2)
6	$(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$	M-them. I / I-them. M (2)
7	$(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$	O-them. M
8	$(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$	O-them. M / O-them. I (2)
9	$(3.1\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ 2.2\ 1.3)$	I-them. O / I-them. M (2)
10	$(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$	I-them. M
11	$(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$	O-them. O
12	$(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2\ 2.3})$	O-them. I
13	$(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$	I-them. O / O-them. I (2)
14	$(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$	I-them. O
15	$(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$	I-them. I

As we recognize, SS15 cannot be written as blocks of n-tomic n-ads (cf. Toth 2008b). Moreover, while in $SRG_{3,3}$ the dual-identical sign class (3.1 2.2 1.3) × (3.1 2.2 1.3) determines the two blocks of three trichotomic triads, in $SRG_{4,3}$, there is no dual-identical sign class. It follows that there is no symmetric $SRG_{4,3}$ model. Nevertheless, a maximal model for $SRG_{4,3}$ displays even amounts of M, O and I thematizates:

Maximal $SRG_{4,3}$max:

Thematized M: 7 (nos. 1, 4, 6, 7, 8, 9, 10)
Thematized O: 7 (nos. 2, 4, 5, 9, 11, 13, 14)
Thematized I: 7 (nos. 3, 5, 6, 8, 12, 13, 15)

As for minimal $SRG_{4,3}$ models, we get two variants. In the first model, we restrict thematizates to the cases appearing after the slash in the thematization alternatives of the above list. In the second model, we restrict thematizates to the cases appearing before the slash in the above thematization alternatives. As it turns out, in both minimal $SRG_{4,3}$ models, we get (2n : n : 2n) correlations of the amounts of M, O and I thematizates:

Minimal $SRG_{4,3}$min1:

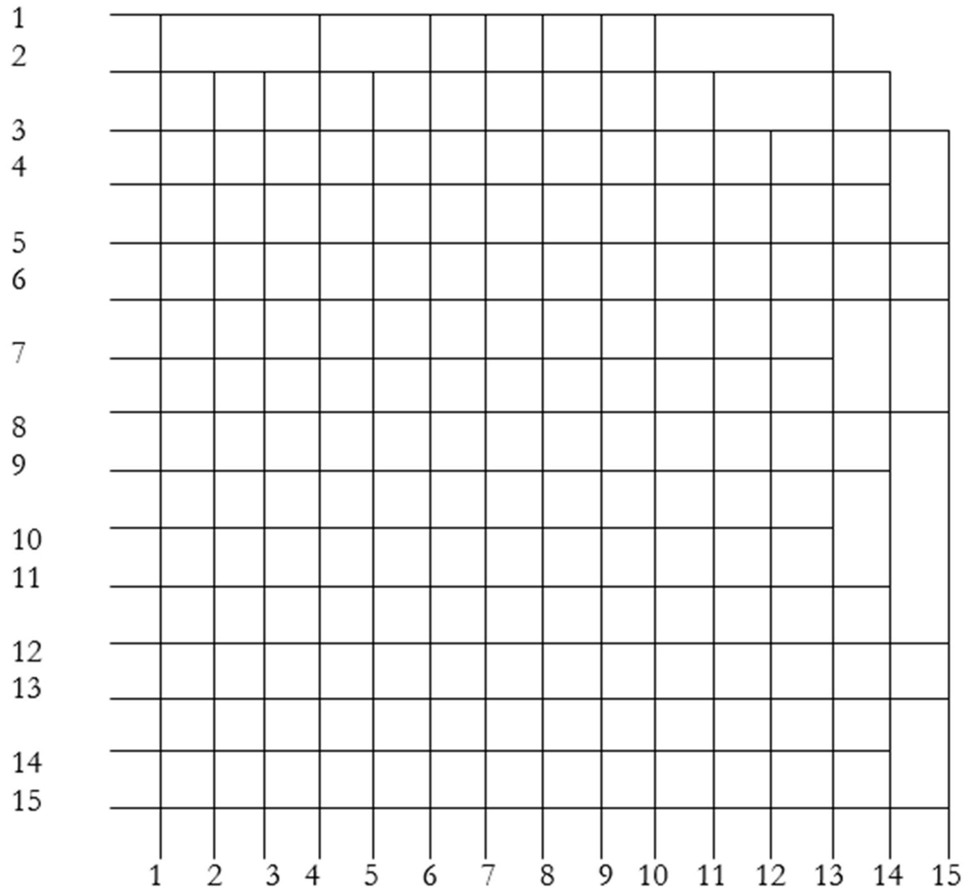
Thematized M: 6 (nos. 1, 4, 6, 7, 9, 10)
Thematized O: 3 (nos. 2, 11, 14)
Thematized I: 6 (nos. 3, 5, 8, 12, 13, 15)

Minimal $SRG_{4,3}$min2:

Thematized M: 4 (nos. 1, 7, 8, 10)
Thematized O: 8 (nos. 2, 4, 5, 9, 11, 12, 13, 14)

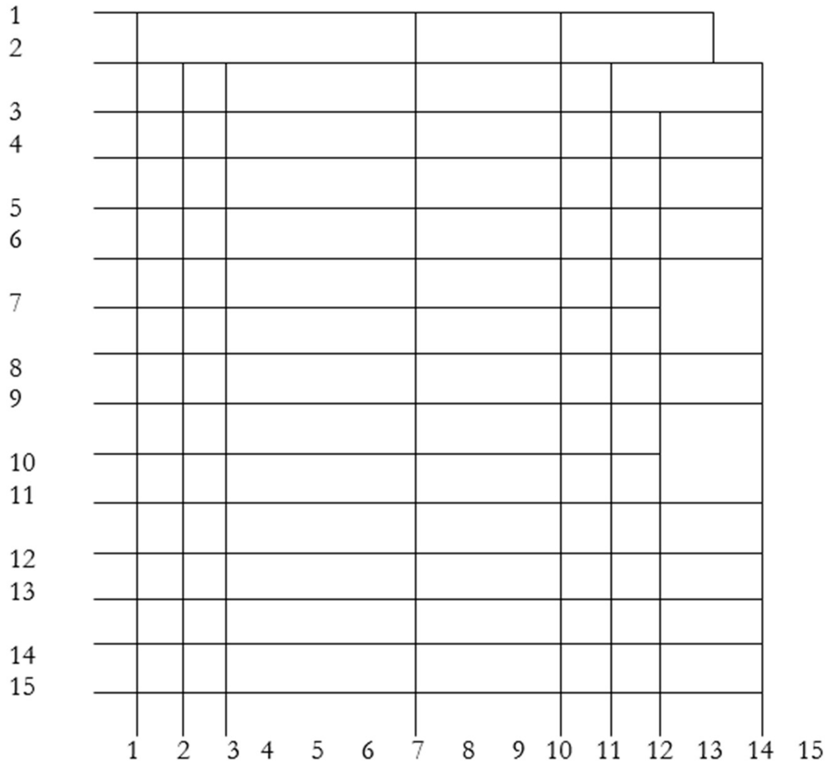
Thematized I: 4 (nos. 3, 6, 12, 15)

The following graph presents $SRG_{4,3}max$. In displays 208 points of intersecting pre-semiotic connections and is thus the maximal pre-semiotic learning space or antimatroid possible over the pre-semiotic sign relation $SR_{4,3}$:

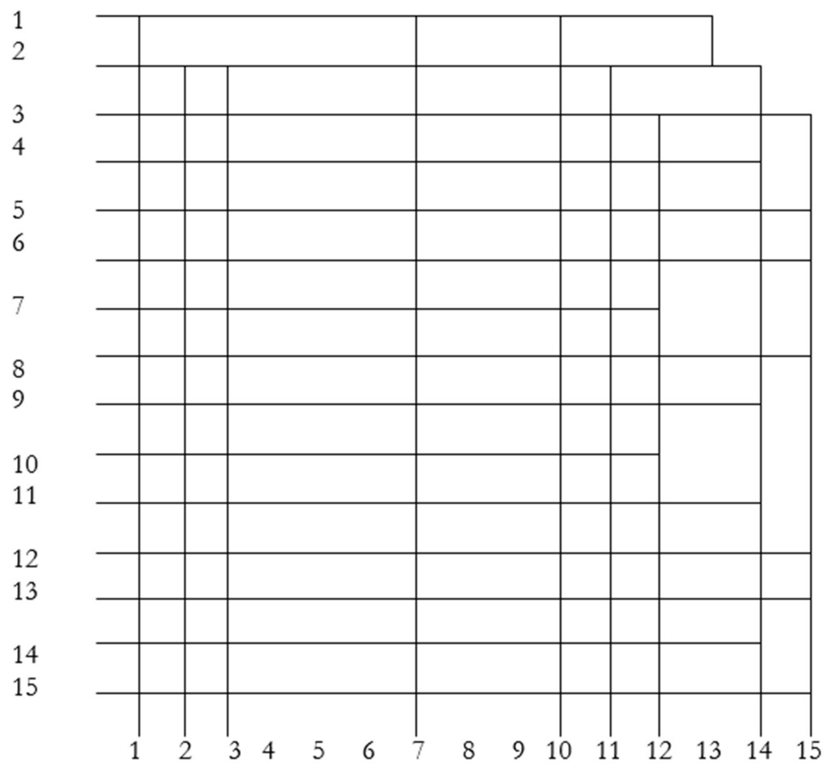


As one recognizes, the rotational "stairwell" structure of $SRG_{3,3}$ appears non-symmetric in $SRG_{4,3}max$.

The graph of $SRG_{4,3}min1$ shows 114 points of intersecting pre-semiotic connections. Like in the graph of $SRG_{4,3}max$, the antimatroidal "stairwell" structure (3-2-1; 3-2-1; 3-2-1) is strongly disturbed. In $SRG_{4,3}min1$, there are also many pre-semiotic connections that bridge over undefined pre-semiotic intersection points:



The graph of SRG$_{4,3}$min2 shows 121 points of intersecting pre-semiotic connections:



As in the graph of $SRG_{4,3}$max and $SRG_{4,min}$1, the antimatroidal "stairwell" structure

3-2-1; 3-2-1; 3-2-1
2-1-3; 2-1-3; 2-1-3
1-3-2; 1-3-2; 1-3-2
3-2-1; 3-2-1; 3-2-1
2-1-3; 2-1-3; 2-1-3
1-3-2; 1-3-2; 1-3-2
3-2-1; 3-2-1; 3-2-1
2-1-3; 2-1-3; 2-1-3
1-3-2; 1-3-2; 1-3-2

is strongly disturbed. As we see, $SRG_{4,3}$min2 differs from $SRG_{4,3}$min1 solely in preserving the thematization quadrant

3-1-2-3—7—10-11-12—14- 15
-
5
6
-
8
-
12
13
-
15

It goes without further demonstration, that none of the three $SRG_{4,3}$ models can be appropriately stratified, since there is not filtration like in $SRG_{3,3}$. Hence, in accordance with our above insights, $SRG_{4,3}$ contains sub-spaces, but the union of the sub-spaces of $SRG_{4,3}$min1 and $SRG_{4,3}$min2 does not yield $SRG_{4,,3}$max.

4. As we did above for the connections between the subsets for ((1,1), (2,1), (3,1), ..., (9,1)) in $SRG_{3,3}$, we will now show some possible pre-semiotic connections in $SRG_{4,3}$. Since permutations of sign relations are the most complex source for semiotic structures (cf. Toth 2008a, pp. 177 ss.; 2008c, pp. 28 ss.), and since the thematization structures are not changed by permutations of reality thematics (Toth 2008d), we show in the following the 4! = 24 possible permutations of the pre-semiotic sign class (3.1 2.1 1.2 0.3) with its dual reality thematic (3.0 2.1 <u>1.2 1.3</u>) and its two structural realities (M-them. O) / (M-them. I). It is thus possible to construct any SRG models and thus any learning spaces using permutated reality thematics instead of "non-permutated" ones. In the following table, the left column displays the permutated reality thematics, the middle column gives the respective structure of the structural reality, and the right column shows the categorial structure of the structural realities:

(1.3 1.2 2.1 3.0)	$(1^{2,>} \rightarrow 2^1 \leftrightarrow 3^1)$	$[[id1, \beta^\circ], [\alpha, \alpha^\circ], [\beta, \gamma^\circ]]$
(2.1 3.0 1.2 1.3)	$(2^1 \leftrightarrow 3^1 \leftarrow 1^{2,<})$	$[[\beta, \gamma^\circ], [\alpha^\circ\beta^\circ, \delta^\circ], [id1, \beta]]$
(1.2 1.3 2.1 3.0)	$(1^{2,<} \rightarrow 2^1 \leftrightarrow 3^1)$	$[[id1, \beta], [\alpha, \alpha^\circ\beta^\circ], [\beta, \gamma^\circ]]$
(1.3 3.0 1.2 2.1)	$(1^{1,>} \rightarrow 3^1 \leftarrow 1^1 \rightarrow 2^1)$	$[[\beta\alpha, \gamma^\circ\delta^\circ], [\alpha^\circ\beta^\circ, \delta^\circ], [\alpha, \alpha^\circ]]$
(1.3 2.1 1.2 3.0)	$(1^{1,>} \rightarrow 2^1 \leftarrow 1^1 \rightarrow 3^1)$	$[[\alpha, \alpha^\circ\beta^\circ], [\alpha^\circ, \alpha], [\beta\alpha, \delta^\circ]]$
(1.2 3.0 1.3 2.1)	$(1^{1,<} \rightarrow 3^1 \leftarrow 1^1 \rightarrow 2^1)$	$[[\beta\alpha, \delta^\circ], [\alpha^\circ\beta^\circ, \delta\gamma], [\alpha, \alpha^\circ\beta^\circ]]$

(2.1 1.3 1.2 3.0)	$(2^1 \leftarrow 1^{2,>} \rightarrow 3^1)$	$[[\alpha^\circ, \beta\alpha], [\mathrm{id}1, \beta^\circ], [\beta\alpha, \delta^\circ]]$
(2.1 3.0 1.3 1.2)	$(2^1 \leftrightarrow 3^1 \leftarrow 1^{2,>})$	$[[\beta, \gamma^\circ], [\alpha^\circ\beta^\circ, \delta\gamma], [\mathrm{id}1, \beta^\circ]]$ (unconnected!)
(1.2 2.1 1.3 3.0)	$(1^{1,<} \rightarrow 2^1 \leftarrow 1^1 \rightarrow 3^1)$	$[[\alpha, \alpha^\circ], [\alpha^\circ, \beta\alpha], [\beta\alpha, \gamma^\circ\delta^\circ]]$
(1.3 3.0 2.1 1.2)	$(1^{1,>} \rightarrow 3^1 \leftrightarrow 2^1 \leftarrow 1^1)$	$[[\beta\alpha, \gamma^\circ\delta^\circ], [\beta^\circ, \gamma], [\alpha^\circ, \alpha]]$
(2.1 1.2 1.3 3.0)	$(2^1 \leftarrow 1^{2,<} \rightarrow 3^1)$	$[[\alpha^\circ, \alpha], [\mathrm{id}1, \beta], [\beta\alpha, \gamma^\circ\delta^\circ]]$
(1.2 3.0 2.1 1.3)	$(1^{1,<} \rightarrow 3^1 \leftrightarrow 2^1 \leftarrow 1^1)$	$[[\beta\alpha, \delta^\circ], [\beta^\circ, \gamma], [\alpha^\circ, \beta\alpha]]$
(1.3 1.2 3.0 2.1)	$(1^{2,>} \rightarrow 3^1 \leftrightarrow 2^1)$	$[[\mathrm{id}1, \beta^\circ], [\beta\alpha, \delta^\circ], [\beta^\circ, \gamma]]$
(3.0 2.1 1.2 1.3)	$(3^1 \leftrightarrow 2^1 \leftarrow 1^{2,<})$	$[[\beta^\circ, \gamma], [\alpha^\circ, \alpha], [\mathrm{id}1, \beta]]$
(1.2 1.3 3.0 2.1)	$(1^{2,<} \rightarrow 3^1 \leftrightarrow 2^1)$	$[[\mathrm{id}1, \beta], [\beta\alpha, \gamma^\circ\delta^\circ], [\beta^\circ, \gamma]]$
(3.0 1.2 1.3 2.1)	$(3^1 \leftarrow 1^{2,<} \rightarrow 2^1)$	$[[\alpha^\circ\beta^\circ, \delta], [\mathrm{id}1, \beta], [\alpha, \alpha^\circ\beta^\circ]]$

(3.0 1.2 1.3 2.1)	$(3^1 \leftarrow 1^{2,<} \rightarrow 2^1)$	$[[\alpha°\beta°, \delta], [id1, \beta], [\alpha, \alpha°\beta°]]$
(1.3 2.1 3.0 1.2)	$(1^{1,>} \rightarrow 2^1 \leftrightarrow 3^1 \leftarrow 1^1)$	$[[\alpha, \alpha°\beta°], [\beta, \gamma°], [\alpha°\beta°, \delta]]$
(3.0 1.3 2.1 1.2)	$(3^1 \leftarrow 1^{1,>} \rightarrow 2^1 \leftarrow 1^1)$	$[[\alpha°\beta°, \delta\gamma], [\alpha, \alpha°\beta°], [\alpha°, \alpha]]$
(2.1 1.3 3.0 1.2)	$(2^1 \leftarrow 1^{1,>} \rightarrow 3^1 \leftarrow 1^1)$	$[[\alpha°, \beta\alpha], [\beta\alpha, \gamma°\delta°], [\alpha°\beta°, \delta]]$
(3.0 2.1 1.3 1.2)	$(3^1 \leftrightarrow 2^1 \leftarrow 1^{2,>})$	$[[\beta°, \gamma], [\alpha°, \beta\alpha], [id1, \beta°]]$
(1.2 2.1 3.0 1.3)	$(1^{1,<} \rightarrow 2^1 \leftrightarrow 3^1 \leftarrow 1^1)$	$[[\alpha, \alpha°], [\beta, \gamma°], [\alpha°\beta°, \delta\gamma]]$

From the above fragment, we also recognize that full information about semiotic and pre-semiotic connections in any (semiotic or pre-semiotic spaces) between reality thematics and their permutations can only be won by using both numerical (or "static") and "dynamic" category theoretic analysis.

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Tetratomic tetrads from an extension of the set of the pre-semiotic sign classes

1. Unlike the "classic" semiotic sign relation $SR_{3,3}$ = (3.a 2.b 1.c), which is triadic-trichotomic, the "trans-classic" pre-semiotic sign relation $SR_{4,3}$ = (3.a 2.b 1.c 0.d) is tetradic-trichotomic. As a tetradic-trichotomic sign relation, $SR_{4,3}$ thus can be considered an expansion of $SR_{3,3}$. However, at the same time, $SR_{4,3}$ is also a fragment of the tetradic-tetratomic sign relation $SR_{4,4}$ (cf. Toth 2007, pp. 214 ss.), which can be seen best if we have a look at the structural realities presented by the reality thematics of the 15 pre-semiotic sign classes:

31 (3.1 2.1 1.1 0.1) × (1.0 $\underline{1.1\ 1.2}$ 1.3)
32 (3.1 2.1 1.1 0.2) × (2.0 $\underline{1.1\ 1.2}$ 1.3)
33 (3.1 2.1 1.1 0.3) × (3.0 $\underline{1.1\ 1.2}$ 1.3)

34 (3.1 2.1 1.2 0.2) × (2.0 $\underline{2.1\ 1.2}$ 1.3)
35 (3.1 2.1 1.2 0.3) × (3.0 $\underline{2.1\ 1.2}$ 1.3)

36 (3.1 2.1 1.3 0.3) × (3.0 $\underline{3.1\ 1.2}$ 1.3)

37 (3.1 2.2 1.2 0.2) × (2.0 $\underline{2.1\ 2.2}$ 1.3)
38 (3.1 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2}$ 1.3)

39 (3.1 2.2 1.3 0.3) × (3.0 $\underline{3.1\ 2.2}$ 1.3)

40 (3.1 2.3 1.3 0.3) × (3.0 $\underline{3.1\ 3.2}$ 1.3)

41 (3.2 2.2 1.2 0.2) × (2.0 $\underline{2.1\ 2.2}$ 2.3)
42 (3.2 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2}$ 2.3)

43 (3.2 2.2 1.3 0.3) × (3.0 $\underline{3.1\ 2.2}$ 2.3)

44 (3.2 2.3 1.3 0.3) × (3.0 $\underline{3.1\ 3.2}$ 2.3)
45 (3.3 2.3 1.3 0.3) × (3.0 $\underline{3.1\ 3.2}$ 3.3)

Thus, the reality thematic of the first trichotomic triad is characterized by (1.1 1.2). It turns out that each of the 15 reality thematics can be embedded into a trichotomic triads characterized by a pair of sub-signs. However, in order to do that, we have to reconstruct a semiotic system whose part SS15 is. As one easily sees, it is not SS35, which is built from the tetradic-tetratomic sign relation $SR_{4,4}$ = (3.a 2.b 1.c 0.d) and the semiotic inclusion order $a \leq b \leq c \leq d$, since in the pre-semiotic system SS15, a, b, c, d ∈ {1, 2, 3} and thus ≠ 0 (cf. Bense 1975, p. 65; Toth 2008a).

In the following table, we reconstruct the lacking reality thematics to build trichotomic triads by asterisk (*, **):

1 (3.1 2.1 1.1 0.1) × (1.0 $\underline{1.1\ 1.2}$ 1.3)
2 (3.1 2.1 1.1 0.2) × (2.0 $\underline{1.1\ 1.2}$ 1.3)
3 (3.1 2.1 1.1 0.3) × (3.0 $\underline{1.1\ 1.2}$ 1.3)

*	(3.1 2.1 1.2 0.1) × (1.0 2.1 1.2 1.3)
4	(3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)
5	(3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)

*	(3.1 2.1 1.3 0.1) × (1.0 3.1 1.2 1.3)
**	(3.1 2.1 1.3 0.2) × (2.0 3.1 1.2 1.3)
6	(3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)

*	(3.1 2.2 1.2 0.1) × (1.0 2.1 2.2 1.3)
7	(3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)
8	(3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)

*	(3.1 2.2 1.3 0.1) × (1.0 3.1 2.2 1.3)
**	(3.1 2.2 1.3 0.2) × (2.0 3.1 2.2 1.3)
9	(3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)

*	(3.1 2.3 1.3 0.1) × (1.0 3.1 3.2 1.3)
**	(3.1 2.3 1.3 0.2) × (2.0 3.1 3.2 1.3)
10	(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)

*	(3.2 2.2 1.2 0.1) × (1.0 2.1 2.2 2.3)
11	(3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)
12	(3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)

*	(3.2 2.2 1.3 0.1) × (1.0 3.1 2.2 2.3)
**	(3.2 2.2 1.3 0.2) × (2.0 3.1 2.2 2.3)
13	(3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)

*	(3.2 2.3 1.3 0.1) × (1.0 3.1 3.2 2.3)
**	(3.2 2.3 1.3 0.2) × (2.0 3.1 3.2 2.3)
14	(3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)

*	(3.3 2.3 1.3 0.1) × (1.0 3.1 3.2 3.3)
**	(3.3 2.3 1.3 0.2) × (2.0 3.1 3.2 3.3)
15	(3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3)

2. Hence, we get 10 trichotomic triads and thus a system of 30 pre-semiotic sign classes (SS30). However, the set SS30\SS15 contains sign classes that are not built according to the inclusion order ($a \le b \le c \le d$), which is valid for SS15. But note that this "violation" of semiotic inclusion touches only trichotomic zeroness, i.e. d, so that SS30 can be characterized by the following pre-semiotic inclusion orders:

$a \le b \le c < d$, e.g. (3.1 2.1 1.2 0.3)
$a \le b \le c = d$, e.g. (3.1 2.1 1.2 0.2)
$a \le b \le c > d$, e.g. (3.1 2.1 1.2 0.1)

Without this constraint that is based on Bense's distinction between relational and categorial numbers (cf. Toth 2008a), the maximal amount of sign classes from $SR_{4,3}$ would be 43 = 64.

Moreover, if we look, e.g. at the reality thematic of the following pre-semiotic dual system:

13 $(3.2\ 2.2\ 1.3\ 0.3) \times (3.0\ \underline{3.1\ 2.2}\ 2.3)$

we recognize that the pair of sub-signs characteristic for the embedding of no. 13 into a trichotomic triads (3.1 2.2) belongs partly to the thematizing and partly to the thematized group of sub-signs in the following structural reality:

$(\underline{3.0\ 3.1}\ \underline{2.2\ 2.3}) \equiv 32 \leftrightarrow 22$,

which thus can be interpreted both as object-thematized interpretant $(32 \leftarrow 22)$ and as interpretant-thematized object $(32 \rightarrow 22)$.

3. We will now order the 30 pre-semiotic sign classes over this extension of $SR_{4,3}$, which we shall call $SR_{4,3}*$, according to their types of thematizations introduced in Toth (2007, pp. 214 ss.).

1. Homogeneous thematizations:

1	$(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$	14
11	$(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$	24
15	$(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$	34

2. Dyadic thematizations:

2	$(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$	$21 \leftarrow 13$
3	$(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$	$31 \leftarrow 13$
4	$(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$	$22 \leftrightarrow 12$
6	$(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$	$32 \leftrightarrow 12$
7	$(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$	$23 \rightarrow 11$
10	$(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$	$33 \rightarrow 11$
*	$(3.2\ 2.2\ 1.2\ 0.1) \times (1.0\ \underline{2.1\ 2.2\ 2.3})$	$11 \leftarrow 23$
12	$(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2\ 2.3})$	$31 \leftarrow 23$
13	$(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$	$32 \leftrightarrow 22$
14	$(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$	$33 \rightarrow 21$
*	$(3.3\ 2.3\ 1.3\ 0.1) \times (1.0\ \underline{3.1\ 3.2\ 3.3})$	$11 \leftarrow 33$
*	$(3.3\ 2.3\ 1.3\ 0.2) \times (2.0\ \underline{3.1\ 3.2\ 3.3})$	$21 \leftarrow 33$

3. Triadic thematizations:

*	$(3.1\ 2.1\ 1.2\ 0.1) \times (1.0\ 2.1\ \underline{1.2\ 1.3})$	$11 \leftrightarrow 21 \leftarrow 12$
5	$(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$	$31 \leftrightarrow 21 \leftarrow 12$
*	$(3.1\ 2.1\ 1.3\ 0.1) \times (1.0\ 3.1\ \underline{1.2\ 1.3})$	$11 \leftrightarrow 31 \leftarrow 12$
*	$(3.1\ 2.1\ 1.3\ 0.2) \times (2.0\ 3.1\ \underline{1.2\ 1.3})$	$21 \leftrightarrow 31 \leftarrow 12$
*	$(3.1\ 2.2\ 1.2\ 0.1) \times (1.0\ \underline{2.1\ 2.2}\ 1.3)$	$11 \leftarrow 22 \rightarrow 11$

8	$(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$	$31 \leftarrow 22 \rightarrow 11$
9	$(3.1\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ 2.2\ 1.3)$	$32 \rightarrow 21 \leftrightarrow 11$
*	$(3.1\ 2.3\ 1.3\ 0.1) \times (1.0\ \underline{3.1\ 3.2}\ 1.3)$	$11 \leftarrow 32 \rightarrow 11$
*	$(3.1\ 2.3\ 1.3\ 0.2) \times (2.0\ \underline{3.1\ 3.2}\ 1.3)$	$21 \leftarrow 32 \rightarrow 11$
*	$(3.2\ 2.2\ 1.3\ 0.1) \times (1.0\ 3.1\ \underline{2.2\ 2.3})$	$11 \leftrightarrow 31 \leftarrow 22$
*	$(3.2\ 2.2\ 1.3\ 0.2) \times (2.0\ 3.1\ \underline{2.2\ 2.3})$	$21 \leftrightarrow 31 \leftarrow 22$
*	$(3.2\ 2.3\ 1.3\ 0.1) \times (1.0\ \underline{3.1\ 3.2}\ 2.3)$	$11 \leftarrow 32 \rightarrow 21$
*	$(3.2\ 2.3\ 1.3\ 0.2) \times (2.0\ \underline{3.1\ 3.2}\ 2.3)$	$21 \leftarrow 32 \rightarrow 21$

4. Tetradic thematizations:

*	$(3.1\ 2.2\ 1.3\ 0.1) \times (\underline{1.0}\ \underline{3.1}\ \underline{2.2}\ \underline{1.3})$	$11 \leftrightarrow 31 \leftrightarrow 21 \leftrightarrow 11$
*	$(3.1\ 2.2\ 1.3\ 0.2) \times (\underline{2.0}\ \underline{3.1}\ \underline{2.2}\ \underline{1.3})$	$21 \leftrightarrow 31 \leftrightarrow 21 \leftrightarrow 11$

We can now group these n-adic thematizations to tetratomic n-ads. It turns out that reality thematics, which present dyadic thematization, can be grouped into 3 tetratomic tetrads:

Tetratomic Tetrads of dyadic thematization

2	$(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$	$21 \leftarrow 13$
3	$(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$	$31 \leftarrow 13$
4	$(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$	$22 \leftrightarrow 12$
6	$(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$	$32 \leftrightarrow 12$
7	$(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$	$23 \rightarrow 11$
10	$(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$	$33 \rightarrow 11$
*	$(3.2\ 2.2\ 1.2\ 0.1) \times (1.0\ \underline{2.1\ 2.2\ 2.3})$	$11 \leftarrow 23$
12	$(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2\ 2.3})$	$31 \leftarrow 23$
13	$(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$	$32 \leftrightarrow 22$
14	$(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$	$33 \rightarrow 21$
*	$(3.3\ 2.3\ 1.3\ 0.1) \times (1.0\ \underline{3.1\ 3.2\ 3.3})$	$11 \leftarrow 33$
*	$(3.3\ 2.3\ 1.3\ 0.2) \times (2.0\ \underline{3.1\ 3.2\ 3.3})$	$21 \leftarrow 33$

Reality thematics, which present dyadic thematization, can be grouped into 3 tetratomic tetrads plus the $SR_{4,3}$-equivalent of the dual-identical sign class (3.1 2.2 1.3) in $SR_{3,3}$:

*	$(3.1\ 2.1\ 1.2\ 0.1) \times (1.0\ 2.1\ \underline{1.2\ 1.3})$	$11 \leftrightarrow 21 \leftarrow 12$
5	$(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$	$31 \leftrightarrow 21 \leftarrow 12$
*	$(3.1\ 2.1\ 1.3\ 0.1) \times (1.0\ 3.1\ \underline{1.2\ 1.3})$	$11 \leftrightarrow 31 \leftarrow 12$
*	$(3.1\ 2.1\ 1.3\ 0.2) \times (2.0\ 3.1\ \underline{1.2\ 1.3})$	$21 \leftrightarrow 31 \leftarrow 12$
*	$(3.1\ 2.2\ 1.2\ 0.1) \times (1.0\ \underline{2.1\ 2.2}\ 1.3)$	$11 \leftarrow 22 \rightarrow 11$
8	$(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$	$31 \leftarrow 22 \rightarrow 11$
*	$(3.1\ 2.3\ 1.3\ 0.1) \times (1.0\ \underline{3.1\ 3.2}\ 1.3)$	$11 \leftarrow 32 \rightarrow 11$
*	$(3.2\ 2.3\ 1.3\ 0.2) \times (2.0\ \underline{3.1\ 3.2}\ 2.3)$	$21 \leftarrow 32 \rightarrow 21$

*	(3.1 2.3 1.3 0.2) $\times$ (2.0 $\underline{3.1\ 3.2}$ 1.3)	$21 \leftarrow 32 \rightarrow 11$
*	(3.2 2.2 1.3 0.1) $\times$ (1.0 3.1 $\underline{2.2\ 2.3}$)	$11 \leftrightarrow 31 \leftarrow 22$
*	(3.2 2.2 1.3 0.2) $\times$ (2.0 3.1 $\underline{2.2\ 2.3}$)	$21 \leftrightarrow 31 \leftarrow 22$
*	(3.2 2.3 1.3 0.1) $\times$ (1.0 $\underline{3.1\ 3.2}$ 2.3)	$11 \leftarrow 32 \rightarrow 21$
9	(3.1 2.2 1.3 0.3) $\times$ ($\underline{3.0\ 3.1}$ 2.2 1.3)	$32 \rightarrow 21 \leftrightarrow 11$

Although the tetradic pre-semiotic sign class (3.1 2.2 1.3 0.3) $\times$ (3.0 3.1 2.2 1.3) is only dual-invariant respecting its triadic part relation (3.1 2.2 1.3), the sign class (3.1 2.2 1.3 0.3) and its reality thematic (3.0 3.1 2.2 1.3) hang together with all other sign classes and reality thematics of this tetratomic tetrad of triadic thematization, respectively, by at least one sub-sign. Thus, the pre-semiotic dual system (3.1 2.2 1.3 0.3) $\times$ (3.0 3.1 2.2 1.3) shares this type of connectedness with the semiotic dual system (3.1 2.2 1.3) $\times$ (3.1 2.2 1.3).

4. In Toth (2008b), we have shown that both the semiotic system SS10 over $SR_{3,3}$ and the pre-semiotic system SS27 over $SR_{3,3}$ with abolishment of the semiotic inclusion order $a \leq b \leq c$ are homeostatic. It thus may astonish that also both SS15 and SS30 over $SR_{4,3}$ are homeostatic, despite their lacking of a (genuine) dual-identical sign class. The reason is the for-mentioned connectedness of the pre-semiotic dual system (3.1 2.2 1.3 0.3) $\times$ (3.0 3.1 2.2 1.3) by at least one sub-sign to all other pre-semiotic dual systems both from SS15 and from SS30:

1	(3.1 2.1 1.1 0.1) × (1.0 1.1 1.2 1.3)
2	(3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3)
3	(3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3)
*	(3.1 2.1 1.2 0.1) × (1.0 2.1 1.2 1.3)
4	(3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)
5	(3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)
*	(3.1 2.1 1.3 0.1) × (1.0 3.1 1.2 1.3)
**	(3.1 2.1 1.3 0.2) × (2.0 3.1 1.2 1.3)
6	(3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)
*	(3.1 2.2 1.2 0.1) × (1.0 2.1 2.2 1.3)
7	(3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)
8	(3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)
*	(3.1 2.2 1.3 0.1) × (1.0 3.1 2.2 1.3)
**	(3.1 2.2 1.3 0.2) × (2.0 3.1 2.2 1.3)
9	**(3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)**
*	(3.1 2.3 1.3 0.1) × (1.0 3.1 3.2 1.3)
**	(3.1 2.3 1.3 0.2) × (2.0 3.1 3.2 1.3)
10	(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)
*	(3.2 2.2 1.2 0.1) × (1.0 2.1 2.2 2.3)
11	(3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)
12	(3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)
*	(3.2 2.2 1.3 0.1) × (1.0 3.1 2.2 2.3)
**	(3.2 2.2 1.3 0.2) × (2.0 3.1 2.2 2.3)
13	(3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)
*	(3.2 2.3 1.3 0.1) × (1.0 3.1 3.2 2.3)
**	(3.2 2.3 1.3 0.2) × (2.0 3.1 3.2 2.3)
14	(3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)
*	(3.3 2.3 1.3 0.1) × (1.0 3.1 3.2 3.3)
**	(3.3 2.3 1.3 0.2) × (2.0 3.1 3.2 3.3)
15	(3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3)

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Semiotic perspectives from Another World

1. According to Toth (2008b, pp. 177 ss.), each sign class showing the basic triadic-trichotomic order (a.b c.d e.f) with a = 3., c = 2., e = 1. and .b ≤ .d < .f can be rewritten as a system of 6 transpositions according to the 6 possible orders of a sign class (3. → 2. → 1.; 3. → 1. → 2; 2. → 3. → 1.; 2. → 1.→ 3.; 1. → 3. → 2.; 1. → 2. → 3.):

(a.b c.d e.f) (c.d e.f a.b)
(a.b e.f c.d) (e.f a.b c.d)
(c.d a.b e.f) (e.f c.d a.b)

The same is true, of course, for the dual reality thematics of each sign class. In this case, the 6 possible orders (1. → 2. → 3.; 2. → 1. → 3; 1. → 3. → 2.; 3. → 1.→ 2.; 2. → 3. → 1.; 3. → 2. → 1.) lead to the following 6 transpositions:

(f.e d.c b.a) (b.a f.e d.c)
(d.c f.e b.a) (d.c b.a f.e)
(f.e b.a d.c) (b.a d.c f.e)

2. If we now compare two random transpositions of a sign class or its reality thematics (but not out of both), f. ex.

(3.1 2.1 1.3)
(1.3 3.1 2.1)

and if we compare this pair of transpositions with the following pair:

(3.1 2.1 1.3)
(1.3 2.1 3.1),

we recognize that in the latter pair the second transposition is a mirror-picture of the first, insofar as it consists of the same sub-signs, but in reverse order, while in the first pair the two transpositions are not mirroring one another. It now turns out that we can order the 6 transpositions in pairs, so that all pairs consist only of transpositions that are mirror-pictures of one another:

1 (3.1 2.1 1.3) 3 (1.3 3.1 2.1) 5 (2.1 1.3 3.1)
2 (1.3 2.1 3.1) 4 (2.1 3.1 1.3) 6 (3.1 1.3 2.1)

Thus, if M stands for the binary operation of mirroring, i.e. inversion of the order of the sub-signs of a sign class or reality thematic (but not of the order of the prime-signs of the sub-signs), we get

M(3.1 2.1 1.3) = (1.3 2.1 3.1)
M(1.3 3.1 2.1) = (2.1 3.1 1.3)
M(2.1 1.3 3.1) = (3.1 1.3 2.1)

Since M(1.3 2.1 3.1) = (3.1 2.1 1.3), we also have MM(3.1 2.1 1.3) = (3.1 2.1 1.3), thus, the semiotic operation of mirroring works like the logical negation operator.

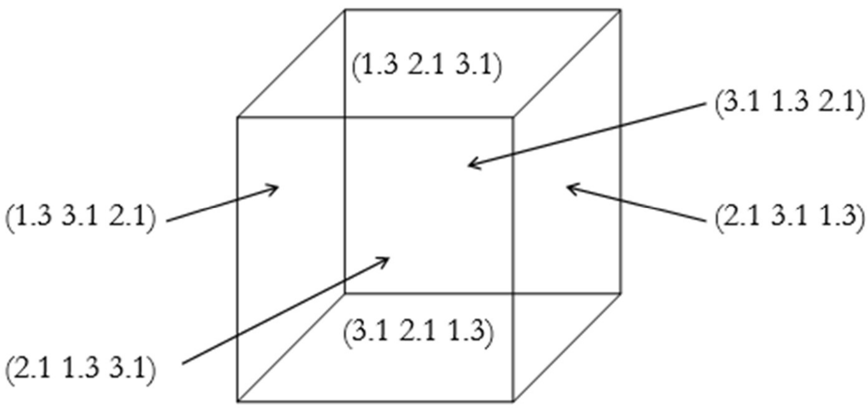
3. In Toth (2008b, pp. 41 ss.), it was shown that the 6 possible reality thematics of each sign class correspond with 6 different system theoretic schemes of observer-standpoints:

(3.1 1.2 1.3)	Objective subject (1), objective subject (2)-thematized subject
(1.3 1.2 3.1)	Objective subject (2), objective subject (1)-thematized subject
(1.2 1.3 3.1)	Objective subject (1), objective subject (2)-thematized subject
(3.1 1.3 1.2)	Objective subject (2), objective subject (1)-thematized subject
(1.3 3.1 1.2)	Objective subject (2), objective subject (1)-thematized subject
(1.2 3.1 1.3)	Objective subject (1), objective subject (2)-thematized subject

Furthermore, the two times 3 seemingly identical types of thematized realities are differentiated according to semiotic priority of what is thematizing or what is thematized (cf. Toth 2008c). For the following table, we use "oS" for objective subject, "sS" for (subjective) subject and "a > b" or "b < a" for "a has semiotic priority to b":

(3.1 1.2 1.3)	sS > (oS1 > oS2)
(1.2 1.3 3.1)	(oS1 > oS2) > sS
(1.2 3.1 1.3)	(oS1 > sS < oS2)
(1.3 1.2 3.1)	(oS2 > oS1) > sS
(3.1 1.3 1.2)	sS > (oS2 > oS1)
(1.3 3.1 1.2)	oS2 > sS < oS1

In other words: The 6 transpositions of a reality thematic and thus of its dual sign class, too, change the system theoretic relationship between subjective subject, objective subject and object and thus the relationship of system and environment in all of the 6 possible standpoints of the observer. Therefore, we are able to visualize the semiotic and system theoretic implications of transpositional reality with the following cube-model in which opposite sides mirror one another:



We may further visualize the inner relationships between the three pairs of mirroring standpoints, or sides of the cube by aid of the semiotic connections of the respective transpositions:

(3.1 2.1 1.3) (1.3 3.1 2.1) (2.1 1.3 3.1)

(1.3 2.1 3.1) (2.1 3.1 1.3) (3.1 1.3 2.1)

If we use category theoretic notation (cf. Toth 2008b, pp. 159 ss.), we may determine exactly the transitions between two opposite mirroring sides or transpositions:

[[β°, id1], [α°, βα]] [[βα, α°β°], [β°, id1]] [[α°, βα], [βα, α°β°]]

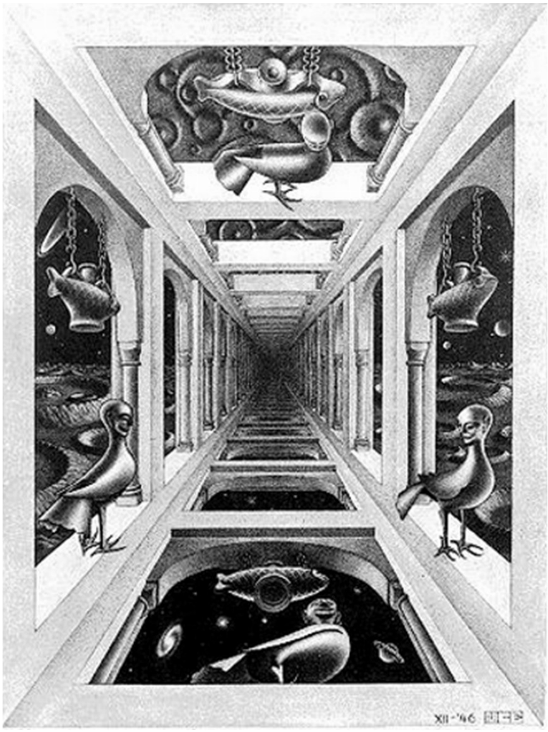
[[α, α°β°], [β, id1]] [[β, id1], [α°β°, βα]] [[α°β°, βα], [α, α°β°]]

We thus get the following three transition classes for the above pairs of transpositions from the left to the right:

[id1]; [βα, α°β°, id1]; [βα,α°β°]

Hence [id1] is the category theoretic-semiotic transition class between below and above, [βα, α°β°, id1] is the respective transition class between in front and at the rear, and [βα, α°β°] is the transition class between the left and the right side of the **semiotic cube** of the transpositions of a sign class or reality thematic. However, this assignment of transpositions to the sides of a cube is arbitrary. Each side of the semiotic cube may be assigned to each of the six transpositions, whereby the only condition is that opposite sides are assigned to the pairs of mirroring transpositions.

4. The cube-model of semiotic transpositions presented above has found a genial anticipation in M.C. Escher's mezzotint "Another World I" (1946) and his woodcut-print "Another World II" (1947). While "Another World II" pictures the cell of a view into "another" world, in "Another World II" the arches continue on as an infinite corridor, thus anticipating the idea of a **semiotic transit-corridor**, which was outlined in Toth (2008a):



M.C. Escher, Other World I, 1946

M.C. Escher, Other World II, 1947

Now, each sign class and thus also each reality thematic hangs together with each other sign class and reality thematic in at least one sub-sign with the dual-inverse sign class (3.1 2.2 1.3). Thus, the 10 sign classes and the 10 reality thematics form a "determinant-symmetric duality system" (Walther 1982). By virtue of this semiotic law, all 6 sides of the semiotic cube depicted above hang together, too, with all transpositions of the 10 sign classes by at least one of the sub-signs of the eigen-real sign class. Hence, if we write each sign class and its transpositions in the form of a semiotic cube, we get a semiotic corridor exactly corresponding to Escher's "Another World I", whereby the arches in Escher's picture, which serve as walls, soils and ceilings at the same time, correspond to the transition classes between the transpositions of each sign class or reality thematic as shown above. Therefore, the semiotic cube is the cell of a semiotic transit-corridor in the sense of the abstract model developed in Toth (2008b).

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Towards a semiotic axiology

1. Under semiotic axiology we do **not** understand a formal contribution to ethics or another pseudo-"science". Rather, this little contribution wants to show a formal device to establish the notion of semiotic value, in addition to the notion of logical value, for the system of triadic semiotics. As it is well known, according to de Saussure (1916), the sign gets its value from the paradigmatic system whose part it is, while the meaning of the sign is part of its syntagmatic structure.

2. We will thus introduce the Saussurean differentiation of syntagm and paradigm into theoretical semiotics. We assign the 6 types of transpositions of each sign class or reality thematic (cf. Toth 2008a, pp. 223 ss.) to the syntagmatic dimension and the 6 possible positions of each transposition to the paradigmatic dimension of "semiology" or semiotics (and not reverse). We may visualize this by the following general scheme:

$\longrightarrow$ positions of transpositions (paradigm)

$\downarrow$

We will further agree, that in their unmarked state, the 6 transpositions of a sign class are mapped onto the 6 possible positions from the left to the right in the following diagram, whereby the transpositions themselves are ordered according to degenerative semiosis both from left to right and from top to bottom, i.e. (3. > 2. > 1. and .3 > .2 > .1):

1	2	3	4	5	6
3.1	3.1	2.1	2.1	1.3	1.3
2.1	1.3	3.1	1.3	3.1	2.1
1.3	2.1	1.3	3.1	2.1	3.1

3. Totally, there are 25 possible combinations of the 6 transpositions of each sign class and reality thematic. Since the combinations are rather tricky, we will briefly sketch them. In the 1st place, all 6 transpositions are possible. In order to achieve a thorough semiotic connection for all of them, we will agree that transposition no. (n+1) must start with the same sub-sign by which the precedent transposition no. (n) has ended. Since the 6 transpositions can be grouped in each 2 beginning with (3.a), with (2.b,) and with (1.c), there are then 2 possibilities in the 2nd place. For example, if (3.1 2.1 1.3) is chosen for the 1st place, then the 2nd place can be assigned with either (1.3 3.1 2.1) or (1.3 2.1 3.1). If the second transposition ends with a sub-sign that has already been used as beginning of a precedent transposition, then there will be only 1 choice left for the 3rd position; otherwise 2 choices, and so on. From the following oversight, we will thus see that there are two main groups of transpositions: such which allow the full cycle of 6 transpositions and such in which the cycle cannot be completed because a new transposition would begin with the third instance of the same sub-sign that had already been used for the beginning of two precedent transpositions, which is impossible.

1. Combinations of (3.1 2.1 1.3)

1	2	3	4	5	6
3.1	1.3	2.1	1.3	3.1	2.1
2.1	3.1	3.1	2.1	1.3	1.3
1.3	2.1	1.3	3.1	2.1	3.1

1	2	3	4	5	6
3.1	1.3	2.1	3.1	2.1	1.3
2.1	3.1	1.3	1.3	3.1	2.1
1.3	2.1	3.1	2.1	1.3	3.1

1	2	3	4	5	6
3.1	1.3	3.1	1.3	2.1	
2.1	2.1	2.1	3.1	3.1	
1.3	3.1	1.3	2.1	1.3 #	

1	2	3	4	5	6
3.1	1.3	3.1	1.3	2.1	
2.1	2.1	2.1	3.1	1.3	
1.3	3.1	1.3	2.1	3.1 #	

2. Combinations of (3.1 1.3 2.1)

1	2	3	4	5	6
3.1	2.1	1.3	2.1	3.1	1.3
1.3	3.1	3.1	1.3	2.1	2.1
2.1	1.3	2.1	3.1	1.3	3.1

1	2	3	4	5	6
3.1	2.1	1.3	3.1	1.3	2.1
1.3	3.1	2.1	2.1	3.1	1.3
2.1	1.3	3.1	1.3	2.1	3.1

1	2	3	4	5	6
3.1	2.1	3.1	1.3	2.1	1.3
1.3	1.3	2.1	3.1	3.1	2.1
2.1	3.1	1.3	2.1	1.3	3.1

1	2	3	4	5	6
3.1	2.1	3.1	1.3		
1.3	1.3	2.1	2.1		
2.1	3.1	1.3	3.1 #		

3. Combinations of (2.1 3.1 1.3)

1	2	3	4	5	6
2.1	1.3	2.1	3.1	1.3	3.1
3.1	3.1	1.3	2.1	2.1	1.3
1.3	2.1	3.1	1.3	3.1	2.1

1	2	3	4	5	6
2.1	1.3	2.1	3.1		
3.1	3.1	1.3	1.3		
1.3	2.1	3.1	2.1 #		

1	2	3	4	5	6
2.1	1.3	3.1	1.3	2.1	3.1
3.1	2.1	2.1	3.1	1.3	1.3
1.3	3.1	1.3	2.1	3.1	2.1

1	2	3	4	5	6
2.1	1.3	3.1	2.1	3.1	1.3
3.1	2.1	1.3	1.3	2.1	3.1
1.3	3.1	2.1	3.1	1.3	2.1

4. Combinations of (2.1 1.3 3.1)

1	2	3	4	5	6
2.1	3.1	1.3	2.1	1.3	3.1
1.3	2.1	3.1	3.1	2.1	1.3
3.1	1.3	2.1	1.3	3.1	2.1

1	2	3	4	5	6
2.1	3.1	1.3	3.1	2.1	1.3
1.3	2.1	2.1	1.3	3.1	3.1
3.1	1.3	3.1	2.1	1.3	2.1

1	2	3	4	5	6
2.1	3.1	2.1	1.3		
1.3	1.3	3.1	3.1		
3.1	2.1	1.3	2.1 #		

1	2	3	4	5	6
2.1	3.1	2.1	1.3	3.1	1.3
1.3	1.3	3.1	2.1	2.1	3.1
3.1	2.1	1.3	3.1	1.3	2.1

5. Combinations of (1.3 3.1 2.1)

1	2	3	4	5	6
1.3	2.1	1.3	3.1		
3.1	3.1	2.1	2.1		
2.1	1.3	3.1	1.3 #		

1	2	3	4	5	6
1.3	2.1	1.3	3.1	2.1	3.1
3.1	3.1	2.1	1.3	1.3	2.1
2.1	1.3	3.1	2.1	3.1	1.3

1	2	3	4	5	6
1.3 3.1 2.1	2.1 1.3 3.1	3.1 2.1 1.3	1.3 2.1 3.1	3.1 1.3 2.1	2.1 3.1 1.3

1	2	3	4	5	6
1.3 3.1 2.1	2.1 1.3 3.1	3.1 1.3 2.1	2.1 3.1 1.3	1.3 2.1 3.1	3.1 2.1 1.3

6. Combinations of (1.3 2.1 3.1)

1	2	3	4	5	6
1.3 2.1 3.1	3.1 2.1 1.3	1.3 3.1 2.1	2.1 3.1 1.3 #		

1	2	3	4	5	6
1.3 2.1 3.1	3.1 2.1 1.3	1.3 2.1 3.1	3.1 1.3 2.1	2.1 3.1 1.3 #	

1	2	3	4	5	6
1.3 2.1 3.1	3.1 2.1 1.3	1.3 2.1 3.1	3.1 1.3 2.1	2.1 1.3 3.1 #	

1	2	3	4	5	6
1.3 2.1 3.1	3.1 1.3 2.1	2.1 3.1 1.3	1.3 3.1 2.1	2.1 1.3 3.1	3.1 2.1 1.3

1	2	3	4	5	6
1.3 2.1 3.1	3.1 1.3 2.1	2.1 1.3 3.1	3.1 2.1 1.3	1.3 3.1 2.1	2.1 3.1 1.3

Herewith all possibilities for the sign class (3.1 2.1 1.3) are exhausted.

4. The only hitherto known semiotic value is the "representation value" that had been introduced into semiotics by Bense (1979). To each sub-sign and sign class or reality thematic a semiotic value is ascribed that is won by addition of the numeric values of the prime-signs. Therefore, in the system of the 10 sign classes, (3.1 2.1 1.1) has a representation value (Rpv) of Rpv = 9, (3.1 2.2 1.3) and (3.2 2.2 1.2) have the same representation value Rpv = 12, and (3.3 2.3 1.3) has Rpv = 15. Therefore, the 10 sign classes can be ordered according to increasing or decreasing Rpv, whereby (3.1 2.1 1.1) has the lowest and (3.3 2.3 1.3) the highest Rpv of the 10 sign classes. Since Bense, already in 1976, had introduced the sign function depending on the two intervals of "semioticity" and "onticity" (Bense 1976, p. 16), we can also say that the sign class with the lowest Rpv has the highest onticity and therefore the lowest semioticity, and the sign class with the highest Rpv has the highest semioticity and thus the lowest onticity.

It is clear, that all 6 transpositions of a sign class and its dual reality thematic have the same representation value. Therefore, the positional semiotic axiology presented in this paper gives a model to further differentiate between the representation values of sign classes by investigating their transpositions. One possible interpretation that we had already introduced in Toth (2008b) is the assignment of semiotic priority to the structural realities presented by the transpostional reality thematics. Since each sign class has only one reality thematic, but 6 different transpositional reality thematics, and since these reality thematics can be ordered by semiotic priority, the semiotic values introduced in this paper can be assigned to them, so that the semiotic values turn out to be positional semiotic measures for semiotic priority.

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Complete and incomplete fuzzy structural realities

1. In the present study, I want to demonstrate by aid of semiotic fuzzy sets, introduced in Toth (2008a, b) that the system of the 10 sign classes is highly fragmentary both from the standpoint of representation and from presentation. The latter can be shown best by comparing the structural realities presented in the reality thematics of the semiotic systems of the 10 and 27 sign classes (SS10; SS27), respectively. The graphs of the respective fuzzy sets, however, do not point to a simple inclusion relation (SS10) $\subset$ (SS27), but towards a type of polycontextural semiotic "inclusion" sketched already in Toth (2003, pp. 54 ss.).

2. If we have a look at SS10, we recognize that it is impossible to order its sign classes according to both increasing ("generative" or "semiosic") interpretant and object relation, but only according to either one. Thus, we get

either	or
(**3.1** 2.1 1.1)	(3.1 **2.1** 1.1)
(**3.1** 2.1 1.2)	(3.1 **2.1** 1.2)
(**3.1** 2.1 1.3)	(3.1 **2.1** 1.3)
(**3.1** 2.2 1.2)	(3.1 **2.2** 1.2)
(**3.1** 2.2 1.3)	(3.1 **2.2** 1.3)
- - - - - - - - - - - - - - -	- - - - - - - - - - - - - - -
(**3.1** 2.3 1.3)	(3.2 **2.2** 1.2)
(**3.2** 2.2 1.2)	(3.2 **2.2** 1.3)
(**3.2** 2.2 1.3)	(3.1 **2.3** 1.3)
(**3.2** 2.3 1.3)	(3.2 **2.3** 1.3)
(**3.3** 2.3 1.3)	(3.2 **2.3** 1.3)

As one sees, the first 5 sign classes are the same in both orderings, but starting with the 6th sign class, one has to decide to order the sign classes either according to their interpretant (left) or their object relation (right). Generally, the same holds true for any ordering of SS10 according to two sign relations (I-O/O-I; I-M/M-I; O-M/M-O). We will formulate this in the form of a semiotic theorem:

Theorem: It is impossible to order all sign classes of SS10 according to more than one sign relation in strictly increasing (generative; semiosic) or strictly decreasing (degenerative; retro-semiosic) order.

However, one also recognizes that the first three sign classes above the dashed line form a Trichotomic Triad (cf. Walther 1981, p. 36), while the other seven sign classes do of course not. Nevertheless, Walther (1982) has shown that SS10 can still be ordered in a system of three Trichotomic Triads, if the following two conditions are fulfilled:

1. The eigenreal sign class (3.1 2.2 1.3) must not be a part of any of the three Trichotomic Triads.

2. The three Trichotomic triads must consist of reality thematics whose dual sign classes are ordered not according to the I- or M-relation, but to the O-relation, whereby the first Trichotomic Triad comprises

only sign classes whose object relation is (2.1), the second Trichotomic Triad only sign classes whose object relation is (2.2), and the third Trichotomic Triad only sign classes whose object relation is (2.3).

Now, since SS10 comprises three sign classes with (2.1) and three sign classes with (2.3), but four sign classes with (2.2), the eigenreal sign class (3.1 2.2 1.3) must non be a part of any of the three Trichotomic Triads, which gives us again condition 1. However, since the eigenreal sign class is connected with any other sign class of SS10 by at least one sub-sign, it is therefore connected with all three Trichotomic Triads. In other words, the drawback that (3.1 2.2 1.3) cannot be part of the three Trichotomic Triads is turned into the benefit that only its position outside of the system of the three Trichotomic Triads enables it to "determine" (Walther 1982) the semiotic "duality system" built up by SS10 and their dual reality thematics.

However, this "benefit" is based solely on the fact that SS10 and the three Trichotomic Triads constructed from it are highly fragmentary. This can be seen best, if we have a look at the first Trichotomic Triad above the dashed line in the above table:

(1.1 1.2 1.3) M-them. M
(2.1 1.2 1.3) M-them. O
(3.1 1.2 1.3) M-them. I

In order to get a complete system of both thematzing and thematized realities, one would await all 27 possible combinations from the following general scheme of semiotic thematization:

({M, O, I})-thematized ({M, O, I}),

hence, f. ex., also structural realities like

*(1.1 2.2 2.3) × *(3.2 2.2 1.1)
(2.1 2.2 2.3) × (3.2 2.2 1.2)
(3.1 2.2 2.3) × (3.2 2.2 1.3),

where the first dual system does not belong to SS10 (marked by asterisk)

or

*(1.1 3.2 3.3) × *(3.3 2.3 1.1)
*(2.1 3.2 3.3) × *(3.3 2.3 1.2)
(3.1 3.2 3.3) × (3.3 2.3 1.3),

where the first two dual systems do not belong either to SS10. Thus, the three Trichotomic Triads constructed from SS10 are not symmetric, and the second two Trichotomic Triads of SS10 do not obey the constructional system of the first one, which reasons thus point out that SS10 is highly fragmentary.

Moreover, if we calculate all 27 possible combinations, it also would turn out that the eigenreal sign class which differs from all other sign classes from SS10 in having a triadic structural reality and thus allowing three and not only one type of thematization:

(1.3, 2.2)-them. (3.1)
(1.3, 3.1)-them. (2.2)
(2.2, 3.1)-them. (1.3)

must be combined with the reality thematics of each of the three Trichotomic triads of SS10, which would result in nine Trichotomic Triads and thus again in 27 sign classes. To be brief, if one takes into account that SS10 is fragmentary from above mentioned reasons, we have no other choice than to substitute SS10 by SS27.

As we will see in next chapter, SS27, in addition, also displays structures of presented realities that are only shown, in SS10, by the structural realities of the eigenreal sign class and by transpositions of sign classes, namely the differentiation between left and right thematization (a, b) as well as "sandwich thematization" (c) and their respective reality structures with inverted order of the thematizing sub-signs (d, e, f), f. ex.

$$\underbrace{\begin{array}{ll} \text{a.} & (3.1\ 2.2\ 1.2) \times (\underline{2.1\ 2.2}\ 1.3) \\ \text{b.} & {*}(3.3\ 2.3\ 1.1) \times {*}(1.1\ \underline{3.2\ 3.3}) \\ \text{c.} & {*}(3.2\ 2.1\ 1.2) \times {*}(\underline{2.1}\ 1.2\ \underline{2.3}) \end{array}}_{\text{Dual systems of SS10/27}} \qquad \underbrace{\begin{array}{ll} \text{d.} & (\underline{2.2\ 2.1}\ 1.3) \times (3.1\ 1.2\ 2.2) \\ \text{e.} & {*}(1.1\ 3.3\ 3.2) \times {*}(2.3\ 3.3\ 1.1) \\ \text{f.} & {*}(2.3\ 1.2\ 2.1) \times {*}(1.2\ 2.1\ 3.2) \end{array}}_{\text{Transpositional Dual systems of SS10/27}}$$

Furthermore, in SS 27, there are several cases of triadic structural realities outside of the context of eigenreality, f. ex.

$${*}(3.1\ 2.2\ 1.1) \times {*}(\underline{1.1}\ \underline{2.2}\ \underline{1.3}),\ \text{i.e.} \quad \left\{ \begin{array}{l} (1.1,\ 2.2)\text{-them. } (1.3) \\ (1.1,\ 1.3)\text{-them. } (2.2) \\ (1.3,\ 2.2)\text{-them. } (1.1), \end{array} \right.$$

generally in all sign classes in SS27 whose trichotomic values are pairwise different, i.e. in all (3.a 2.b 1.c) with $a \neq b \neq c$.

Because of the mentioned structures of presented realities that show types that do not occur in the usual display of SS10, for the reality thematics and thus for the structural realities of SS27, we find

SS 10 $\not\subset$ SS27,

although for the dual sign classes, SS10 $\subset$ SS27 holds true. This "paradox" situation shows that purely formal duality does not hold true for reality thematics, which thus apparently transcend purely syntactic logic. As we already pointed out, SS10 is not a sub-set of SS27, but a morphogrammatic fragment (cf. Toth 2003, pp. 54 ss.), which proves that we have to deal here with a polycontextural feature of theoretical semiotics and thus of qualitative-mathematical semiotics.

3. Before the background of the above statements, we can now order the sign classes and reality thematics of SS27 in Trichotomic Triads according to both increasing I- and O- sign relation:

$$\left.\begin{array}{l} 1.\ (3.1\ 2.1\ 1.1) \times (1.1\ \underline{1.2\ 1.3}) \\ 2.\ (3.1\ 2.1\ 1.2) \times (2.1\ \underline{1.2\ 1.3}) \\ 3.\ (3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3}) \end{array}\right\} \text{TrTr1}$$

$$\left.\begin{array}{l} 4.\ {}^*(3.1\ 2.2\ 1.1) \times {}^*(\underline{1.1}\ 2.2\ \underline{1.3}) \\ 5.\ (3.1\ 2.2\ 1.2) \times (\underline{2.1\ 2.2}\ 1.3) \\ 6.\ (3.1\ 2.2\ 1.3) \times (\underline{3.1}\ \underline{2.2}\ \underline{1.3}) \end{array}\right\} \text{TrTr2}$$

$$\left.\begin{array}{l} 7.\ {}^*(3.1.\ 2.3\ 1.1) \times {}^*(\underline{1.1}\ 3.2\ \underline{1.3}) \\ 8.\ {}^*(3.1\ 2.3\ 1.2) \times {}^*(\underline{2.1}\ \underline{3.2}\ \underline{1.3}) \\ 9.\ (3.1\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 1.3) \end{array}\right\} \text{TrTr3}$$

$$\left.\begin{array}{l} 10.\ {}^*(3.2\ 2.1\ 1.1) \times {}^*(\underline{1.1\ 1.2}\ 2.3) \\ 11.\ {}^*(3.2\ 2.1\ 1.2) \times {}^*(\underline{2.1}\ 1.2\ \underline{2.3}) \\ 12.\ {}^*(3.2\ 2.1\ 1.3) \times {}^*(\underline{3.1}\ \underline{1.2}\ \underline{2.3}) \end{array}\right\} \text{TrTr4}$$

$$\left.\begin{array}{l} 13.\ {}^*(3.2\ 2.2\ 1.1) \times {}^*(1.1\ \underline{2.2\ 2.3}) \\ 14.\ (3.2\ 2.2\ 1.2) \times (2.1\ \underline{2.2\ 2.3}) \\ 15.\ (3.2\ 2.2\ 1.3) \times (3.1\ \underline{2.2\ 2.3}) \end{array}\right\} \text{TrTr5}$$

$$\left.\begin{array}{l} 16.\ {}^*(3.2\ 2.3\ 1.1) \times {}^*(\underline{1.1}\ \underline{3.2}\ \underline{2.3}) \\ 17.\ {}^*(3.2\ 2.3\ 1.2) \times {}^*(\underline{2.1}\ 3.2\ \underline{2.3}) \\ 18.\ (3.2\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 2.3) \end{array}\right\} \text{TrTr6}$$

$$\left.\begin{array}{l} 19.\ {}^*(3.3\ 2.1\ 1.1) \times {}^*(\underline{1.1\ 1.2}\ 3.3) \\ 20.\ {}^*(3.3\ 2.1\ 1.2) \times {}^*(\underline{2.1}\ \underline{1.2}\ \underline{3.3}) \\ 21.\ {}^*(3.3\ 2.1\ 1.3) \times {}^*(\underline{3.1}\ 1.2\ \underline{3.3}) \end{array}\right\} \text{TrtR7}$$

$$\left.\begin{array}{l} 22.\ {}^*(3.3\ 2.2\ 1.1) \times {}^*(\underline{1.1}\ \underline{2.2}\ \underline{3.3}) \\ 23.\ {}^*(3.3\ 2.2\ 1.2) \times {}^*(\underline{2.1\ 2.2}\ 3.3) \\ 24.\ {}^*(3.3\ 2.2\ 1.3) \times {}^*(\underline{3.1}\ 2.2\ \underline{3.3}) \end{array}\right\} \text{TrTr8}$$

$$\left.\begin{array}{l} 25.\ {}^*(3.3\ 2.3\ 1.1) \times {}^*(1.1\ \underline{3.2\ 3.3}) \\ 26.\ {}^*(3.3\ 2.3\ 1.2) \times {}^*(2.1\ \underline{3.2\ 3.3}) \\ 27.\ (3.3\ 2.3\ 1.3) \times (3.1\ \underline{3.2\ 3.3}) \end{array}\right\} \text{TrTr9}$$

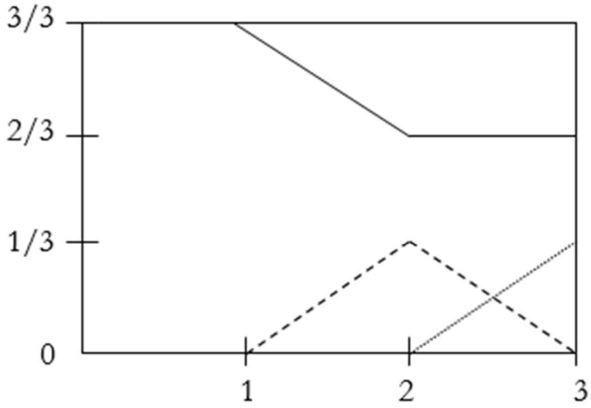
As one easily sees, it is possible to order the dual systems of SS27 according to any pair of sign relations.

4. In this last chapter, we can now finally demonstrate the fragmentarism of SS10 compared to SS27 by aid of semiotic fuzzy sets. We will draw the graphs for all reality thematics of SS27 and mark the dual-systems that do not belong to SS10 again by asterisk. As one sees without any further comment, the main result is that most of the following fuzzy graphs could not even been drawn, since most of the respective trichotomic triads do simply not exist in SS10.

4.1. TrTr1

1. $(3.1\ 2.1\ 1.1) \times (1.1\ \underline{1.2\ 1.3})$
2. $(3.1\ 2.1\ 1.2) \times (2.1\ \underline{1.2\ 1.3})$
3. $(3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3})$

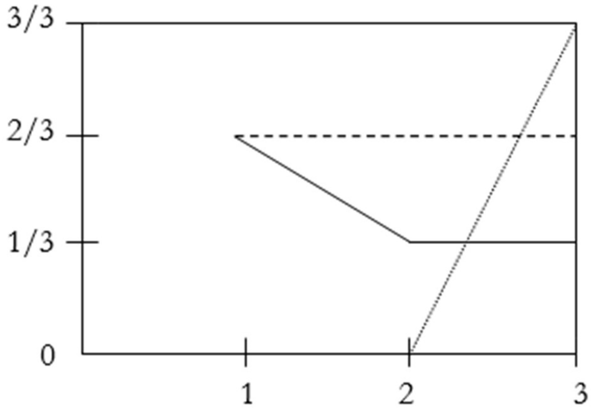
Semiotic fuzzy set for TrTr1



4.2. TrTr2

4. *(3.1 2.2 1.1) × *($\underline{1.1}$ 2.2 $\underline{1.3}$)
5. (3.1 2.2 1.2) × ($\underline{2.1\ 2.2}$ 1.3)
6. (3.1 2.2 1.3) × ($\underline{3.1}$ $\underline{2.2}$ $\underline{1.3}$)

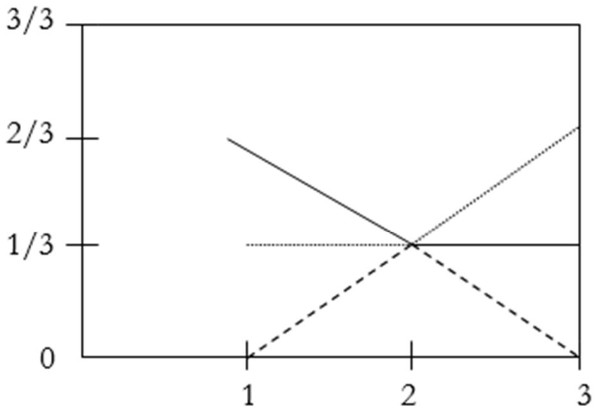
Semiotic fuzzy set for TrTr2



4.3. TrTr3

7. *(3.1. 2.3 1.1) × *($\underline{1.1}$ 3.2 $\underline{1.3}$)
8. *(3.1 2.3 1.2) × *($\underline{2.1}$ $\underline{3.2}$ $\underline{1.3}$)
9. (3.1 2.3 1.3) × ($\underline{3.1\ 3.2}$ 1.3)

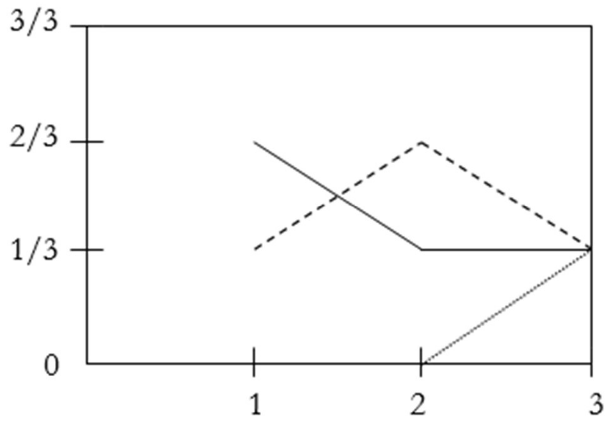
Semiotic fuzzy set for TrTr3



4.4. TrTr4

10. *(3.2 2.1 1.1) × *($\underline{1.1\ 1.2}$ 2.3)
11. *(3.2 2.1 1.2) × *($\underline{2.1}$ 1.2 $\underline{2.3}$)
12. *(3.2 2.1 1.3) × *($\underline{3.1}$ $\underline{1.2}$ $\underline{2.3}$)

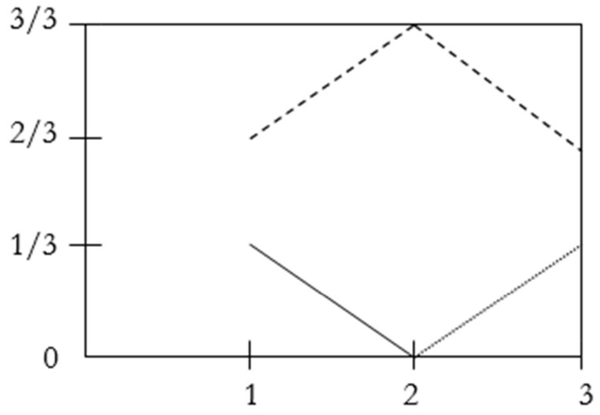
Semiotic fuzzy set for TrTr4



4.5. TrTr5

13. $*(3.2\ 2.2\ 1.1) \times *(1.1\ \underline{2.2\ 2.3})$
14. $(3.2\ 2.2\ 1.2) \times (2.1\ \underline{2.2\ 2.3})$
15. $(3.2\ 2.2\ 1.3) \times (3.1\ \underline{2.2\ 2.3})$

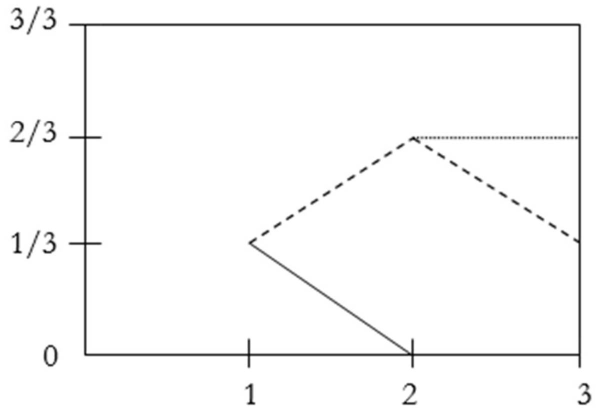
Semiotic fuzzy set for TrTr5



4.6. TrTr6

16. $*(3.2\ 2.3\ 1.1) \times *(\underline{1.1}\ \underline{3.2}\ \underline{2.3})$
17. $*(3.2\ 2.3\ 1.2) \times *(\underline{2.1}\ 3.2\ \underline{2.3})$
18. $(3.2\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 2.3)$

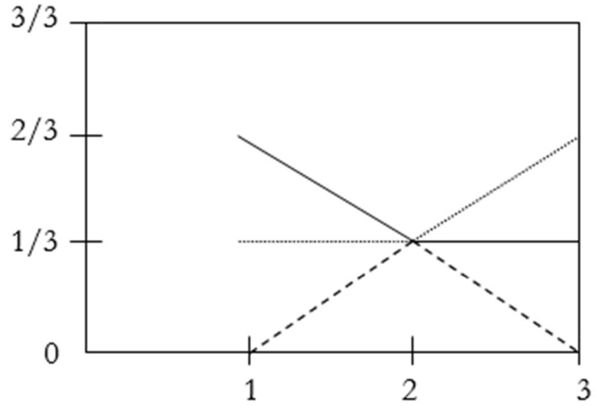
Semiotic fuzzy set for TrTr6



4.7. TrTr7

19. *(3.3 2.1 1.1) × *($\underline{1.1\ 1.2}$ 3.3)
20. *(3.3 2.1 1.2) × *($\underline{2.1}$ $\underline{1.2}$ $\underline{3.3}$)
21. *(3.3 2.1 1.3) × *($\underline{3.1}$ 1.2 $\underline{3.3}$)

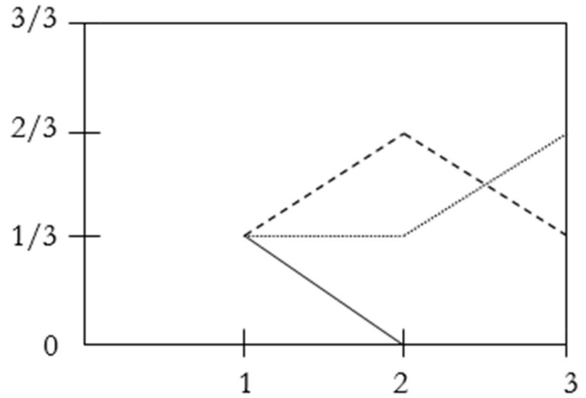
Semiotic fuzzy set for TrTr7



4.8. TrTr8

22. *(3.3 2.2 1.1) × *($\underline{1.1}$ $\underline{2.2}$ $\underline{3.3}$)
23. *(3.3 2.2 1.2) × *($\underline{2.1\ 2.2}$ 3.3)
24. *(3.3 2.2 1.3) × *($\underline{3.1}$ 2.2 $\underline{3.3}$)

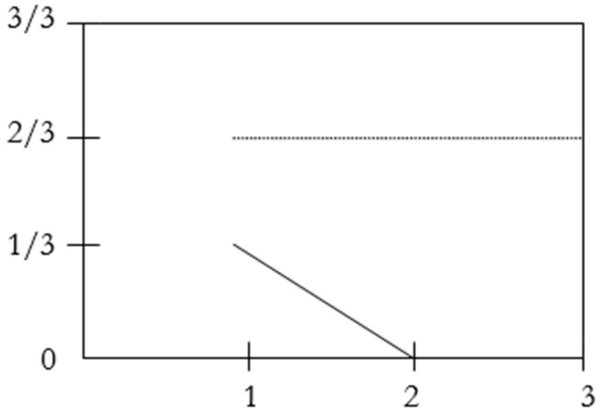
Semiotic fuzzy set for TrTr8



4.9. TrTr9

25. *(3.3 2.3 1.1) × *(1.1 <u>3.2 3.3</u>)
26. *(3.3 2.3 1.2) × *(2.1 <u>3.2 3.3</u>)
27. (3.3 2.3 1.3) × (3.1 <u>3.2 3.3</u>)

Semiotic fuzzy set for TrTr9



The above displayed graphs for semiotic fuzzy sets show that the semiotic reality of SS10 is only a small morphogrammatic fragment of the complete semiotic reality of SS27. Therefore, the above graphs also show that the range of the membership functions of semiotic realities in SS27 is much wider than in SS10 and thus "scoops out" maximally the semiotic continuum of **possible** realities most of which are, however, unrealized in SS10.

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Priority in thematized realities

1. In the system of the 10 sign classes their dual realities contain two types of thematizations, a dyadic and a triadic one:

1. (3.1 2.1 1.3) × (3.1 $\underline{\text{1.2 1.3}}$) M-them. I, i.e. (1.2, 1.3)-them. (3.1)

2. (3.1 2.2 1.3) ×
- ($\underline{\text{3.1 2.2}}$ 1.3) I, O-them. M, i.e. (3.1, 2.2)-them. (1.3)
- (3.1 $\underline{\text{2.2 1.3}}$) O, M-them. I, i.e. (2.2, 1.3)-them. (3.1)
- ($\underline{\text{3.1}}$ 2.2 $\underline{\text{1.3}}$) I, M-thematized O, i.e. (3.1, 1.3)-thematized (2.2)

2. However, as the three types of thematizations presented in the reality thematics of the sign class (3.1 2.2 1.3) show, these two kinds of structural realities appear to be fragmentary from the point of view of their respective types of thematizations. If we abolish the Law of Inclusive Trichotomic order (cf. Toth 2008a), we get a system of 27 sign classes[2] and reality thematics that show the following types of thematizations:

(3.1 2.1 1.1) × (1.1 $\underline{\text{1.2 1.3}}$) M-them. M
(3.2 2.2 1.2) × (2.1 $\underline{\text{2.2 2.3}}$) O-them. O
(3.3 2.3 1.3) × (3.1 $\underline{\text{3.2 3.3}}$) I-them. I

There seems to be only one type of homogenous thematization per triadic sign value, although from a purely structural viewpoint, it is not clear, if the correct structural realities are (a.b $\underline{\text{c.d e.f}}$), ($\underline{\text{a.b c.d}}$ e.f) or ($\underline{\text{a.b}}$ c.d $\underline{\text{e.f}}$) as f.ex. in (2.1 $\underline{\text{1.2 1.3}}$), ($\underline{\text{3.1 3.2}}$ 2.3) and ($\underline{\text{3.1}}$ $\underline{\text{2.2}}$ $\underline{\text{1.3}}$).

(3.1 2.2 1.2) × ($\underline{\text{2.1 2.2}}$ 1.3) O-them. M
*(3.2 2.2 1.1) × (1.1 $\underline{\text{2.2 2.3}}$) O-them. M
*(3.2 2.1 1.2) × ($\underline{\text{2.1}}$ 1.2 $\underline{\text{2.3}}$) O-them. M

In this and the next "trichotomic triads" we recognize now our three alternative structures ($\underline{\text{a.b c.d}}$ e.f), (a.b $\underline{\text{c.d e.f}}$) and ($\underline{\text{a.b}}$ c.d $\underline{\text{e.f}}$), which we shall call left-, right- and sandwich-thematizations (cf. Toth 2007, p. 179):

(3.1 2.3 1.3) × ($\underline{\text{3.1 3.2}}$ 1.3) I-them. M
*(3.3 2.3 1.1) × (1.1 $\underline{\text{3.2 3.3}}$) I-them. M
*(3.3 2.1 1.3) × ($\underline{\text{3.1}}$ 1.2 $\underline{\text{3.3}}$) I-them. M

[2] The sign classes of the complementary set, which does not comprise the set of the 10 sign classes, are marked by asterisk.

$(3.1\ 2.1\ 1.2) \times (2.1\ \underline{1.2\ 1.3})$	M-them. O
$*(3.2\ 2.1\ 1.1) \times (\underline{1.1\ 1.2}\ 2.3)$	M-them. O
$*(3.1\ 2.2\ 1.1) \times (\underline{1.1}\ 2.2\ \underline{1.3})$	M-them. O
$(3.2\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 2.3)$	I-them. O
$*(3.3\ 2.3\ 1.2) \times (2.1\ \underline{3.2\ 3.3})$	I-them. O
$*(3.3\ 2.2\ 1.3) \times (\underline{3.1}\ 2.2\ \underline{3.3})$	I-them. O
$(3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3})$	M-them. I
$*(3.3\ 2.1\ 1.1) \times (\underline{1.1\ 1.2}\ 3.3)$	M-them. I
$*(3.1\ 2.3\ 1.1) \times (\underline{1.1}\ 3.2\ \underline{1.3})$	M-them. I
$(3.2\ 2.2\ 1.3) \times (3.1\ \underline{2.2\ 2.3})$	O-them I
$*(3.2\ 2.3\ 1.2) \times (\underline{2.1}\ 3.2\ \underline{2.3})$	O-them I
$*(3.3\ 2.2\ 1.2) \times (\underline{2.1\ 2.2}\ 3.3)$	O-them I

To our surprise, in the system of the 27 sign classes, we find no less than 6 triadic structural realities:

$*(3.2\ 2.3\ 1.1) \times (\underline{1.1}\ \underline{3.2}\ \underline{2.3})$	triadic
$*(3.3\ 2.2\ 1.1) \times (\underline{1.1}\ \underline{2.2}\ \underline{3.3})$	triadic
$*(3.1\ 2.3\ 1.2) \times (\underline{2.1}\ \underline{3.2}\ \underline{1.3})$	triadic
$*(3.3\ 2.1\ 1.2) \times (\underline{2.1}\ \underline{1.2}\ \underline{3.3})$	triadic
$(3.1\ 2.2\ 1.3) \times (\underline{3.1}\ \underline{2.2}\ \underline{1.3})$	triadic
$*(3.2\ 2.1\ 1.3) \times (\underline{3.1}\ \underline{1.2}\ \underline{2.3})$	triadic

3. However, the three types of structural realities shown above, seem to be fragmentary, too, since by aid of combinatorics we get the following 6 types of thematizations which we will show using the sign class (3.1 2.1 1.3):

$(3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3})$
$(2.1\ 3.1\ 1.3) \times (3.1\ \underline{1.3\ 1.2})$

$(3.1\ 1.3\ 2.1) \times (\underline{1.2}\ 3.1\ \underline{1.3})$
$(2.1\ 1.3\ 3.1 \times (\underline{1.3}\ 3.1\ \underline{1.2})$

$(1.3\ 3.1\ 2.1) \times (\underline{1.2\ 1.3}\ 3.1)$
$(1.3\ 2.1\ 3.1) \times (\underline{1.3\ 1.2}\ 3.1)$

By dualizing the reality thematics which present the respective structural realities, we thus do not get proper sign classes, but transpositions of sign classes which we have proven to be defined sign classes, too (cf. Toth 2008b). Conversely, by starting with sign classes in which the Law of Degenerative Triadic

Order is abolished, i.e. in allowing all 5 transpositions per sign class, we get reality thematics, whose structural realities introduce a new notion into semiotics, namely that of **semiotic priority** amongst which the following two basic types can be distinguished (still using our above example):

1. Priority of (1.2) before (1.3) or reverse
2. Priority of (1.2 1.3) / (1.3 1.2) before (3.1) or reverse
3.

Therefore, type 1 shows priority between the thematizing sub-signs, while type 2 shows priority of between thematizing vs. thematized sub-signs.

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Rough-fuzzy hybridization in semiotics

1. In a former study (Toth 2008), I had introduced rough sets into mathematical semiotics. As one recalls, a rough set is a formal approximation of a crisp set in terms of a pair of sets, which give the lower and the upper approximation of the original set. The lower and upper approximation sets themselves are crisp sets in the standard version of rough set theory (Pawlak 1991), but in other variations, the approximating sets may be fuzzy sets as well. The latter theory is called "rough-fuzzy hybridization" and is the topic of the present study.

2. Sign classes and reality thematics can be compared respecting their representation values (cf. Bense 1981, pp. 86 ss.), respecting their thematized and respecting their thematizing realities (cf. Bense 1981, pp. 111 ss.). All three criteria are ambiguous, since there is no bijective mapping between sign classes or reality thematics onto either representation values, thematized or thematizing realities. In Toth (2008), we had shown that the mapping of the 10 classical and the 27 trans-classical sign classes onto the system of the representation values leads to rough sets. In this study, we will map the system of the representation values onto the thematized and realities of both the classical and the trans-classical semiotic systems and show that we have here a case of rough-fuzzy semiotic hybridization.

3. The following table gives the 10 sign classes (SS10), their representation values and their structural realities, whereby the thematized realities are focussed:

SS10:

1. $(3.1\ 2.1\ 1.1) \times (1.1\ \underline{1.2\ 1.3})$	Rpv = 9	M-them. M
2. $(3.1\ 2.1\ 1.2) \times (2.1\ \underline{1.2\ 1.3})$	Rpv = 10	M-them. O
3. $(3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3})$	Rpv = 11	M-them. I
4. $(3.1\ 2.2\ 1.2) \times (\underline{2.1\ 2.2}\ 1.3)$	Rpv = 11	O-them. M
5. $(3.1\ 2.2\ 1.3) \times (\underline{3.1}\ \underline{2.2}\ \underline{1.3})$	Rpv = 12	(triadic reality*)
6. $(3.1\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 1.3)$	Rpv = 13	I-them. M
7. $(3.2\ 2.2\ 1.2) \times (2.1\ \underline{2.2\ 2.3})$	Rpv = 12	O-them. O
8. $(3.2\ 2.2\ 1.3) \times (3.1\ \underline{2.2\ 2.3})$	Rpv = 13	O-them. I
9. $(3.2\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 2.3)$	Rpv = 14	I-them. O
10. $(3.3\ 2.3\ 1.3) \times (3.1\ \underline{3.2\ 3.3})$	Rpv = 15	I-them. I

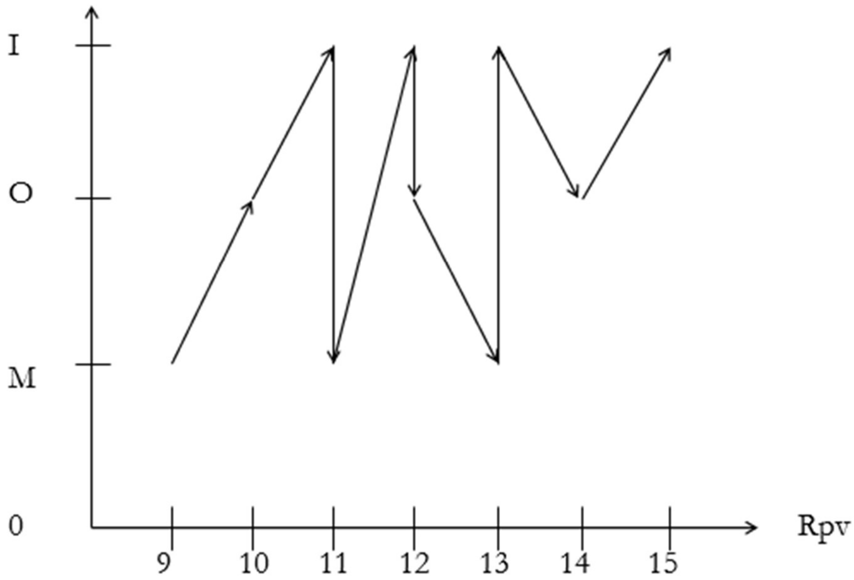
Let therefore DS1 ... DS 10 be the set of objects X = $\{O_1, ..., O_{10}\}$, i.e. the elements of SS10, Rpv = 9 ... Rpv = 15 the first set of attributes $P_i = \{P_1, ..., P_7\}$, and the structural realities the second set of P_j = {M, O, I}. By combining these two sets of attributes we thus have an instance of a combination of fuzzy and rough sets (cf. Dubois and Prader 1992). For the sake of simplification, we will choose the structural reality of (M, O)-them. I from the triadic reality marked above by asterisk (*). Then we can build the following equivalence classes of SS10:

7 equivalence classes of P_i: 3 equivalence classes of P_j:

$$\begin{cases} \{O_1\} \\ \{O_2\} \\ \{O_3, O_4\} \\ \{O_5, O_7\} \\ \{O_6, O_8\} \\ \{O_9\} \\ \{O_{10}\} \end{cases} \qquad \begin{cases} \{O_1, O_4, O_6\} \\ \{O_2, O_7, O_9\} \\ \{O_3, O_5, O_8, O_{10}\} \end{cases}$$

Thus, in the attribute set P_i, the two objects in the third, fourth and fifth equivalence classes, and in P_j, the three objects in the first and in the second and the four objects in the third equivalence classes are indiscernible. We may visualize this recognition in the following rough-fuzzy semiotic graph:

Thematized realities (SS10)



We shall now have a look at the 27 sign classes of SS27, their representation values and their structural realities, whereby again the thematized realities are focussed:

SS27:

1. $(3.1\ 2.1\ 1.1) \times (1.1\ \underline{1.2\ 1.3})$	Rpv = 9	M-them. M
2. $(3.1\ 2.1\ 1.2) \times (2.1\ \underline{1.2\ 1.3})$	Rpv = 10	M-them. O
3. $(3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3})$	Rpv = 11	M-them. I
4. $*(3.1\ 2.2\ 1.1) \times *(\underline{1.1}\ 2.2\ \underline{1.3})$	Rpv = 10	M-them. O
5. $(3.1\ 2.2\ 1.2) \times (\underline{2.1\ 2.2}\ 1.3)$	Rpv = 11	O-them. M
6. $(3.1\ 2.2\ 1.3) \times (\underline{3.1}\ \underline{2.2}\ \underline{1.3})$	Rpv = 12	(triadic reality*)
7. $*(3.1.\ 2.3\ 1.1) \times *(\underline{1.1}\ 3.2\ \underline{1.3})$	Rpv = 11	M-them. I

8. $*(3.1\ 2.3\ 1.2) \times *(\underline{2.1}\ \underline{3.2}\ \underline{1.3})$	Rpv = 12	(triadic reality*)
9. $(3.1\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 1.3)$	Rpv = 13	I-them. M
10. $*(3.2\ 2.1\ 1.1) \times *(\underline{1.1\ 1.2}\ 2.3)$	Rpv = 10	M-them. O
11. $*(3.2\ 2.1\ 1.2) \times *(\underline{2.1}\ 1.2\ \underline{2.3})$	Rpv = 11	O-them. M
12. $*(3.2\ 2.1\ 1.3) \times *(\underline{3.1}\ \underline{1.2}\ \underline{2.3})$	Rpv = 12	(triadic reality *)
13. $*(3.2\ 2.2\ 1.1) \times *(1.1\ \underline{2.2\ 2.3})$	Rpv = 11	O-them. M
14. $(3.2\ 2.2\ 1.2) \times (2.1\ \underline{2.2\ 2.3})$	Rpv = 12	O-them. O
15. $(3.2\ 2.2\ 1.3) \times (3.1\ \underline{2.2\ 2.3})$	Rpv = 13	O-them. I
16. $*(3.2\ 2.3\ 1.1) \times *(\underline{1.1}\ \underline{3.2}\ \underline{2.3})$	Rpv = 12	(triadic reality*)
17. $*(3.2\ 2.3\ 1.2) \times *(\underline{2.1}\ 3.2\ \underline{2.3})$	Rpv = 13	O-them. I
18. $(3.2\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 2.3)$	Rpv = 14	I-them. O
19. $*(3.3\ 2.1\ 1.1) \times *(\underline{1.1\ 1.2}\ 3.3)$	Rpv = 11	M-them. I
20. $*(3.3\ 2.1\ 1.2) \times *(\underline{2.1}\ \underline{1.2}\ \underline{3.3})$	Rpv = 12	(triadic reality*)
21. $*(3.3\ 2.1\ 1.3) \times *(\underline{3.1}\ 1.2\ \underline{3.3})$	Rpv = 13	I-them. M
22. $*(3.3\ 2.2\ 1.1) \times *(\underline{1.1}\ \underline{2.2}\ \underline{3.3})$	Rpv = 12	(triadic reality*)
23. $*(3.3\ 2.2\ 1.2) \times *(\underline{2.1\ 2.2}\ 3.3)$	Rpv = 13	O-them. I
24. $*(3.3\ 2.2\ 1.3) \times *(\underline{3.1}\ 2.2\ \underline{3.3})$	Rpv = 14	I-them. O
25. $*(3.3\ 2.3\ 1.1) \times *(1.1\ \underline{3.2\ 3.3})$	Rpv = 13	I-them. M
26. $*(3.3\ 2.3\ 1.2) \times *(2.1\ \underline{3.2\ 3.3})$	Rpv = 14	I-them. O
27. $(3.3\ 2.3\ 1.3) \times (3.1\ \underline{3.2\ 3.3})$	Rpv = 15	I-them. I

In SS27, we can build the following equivalence classes:

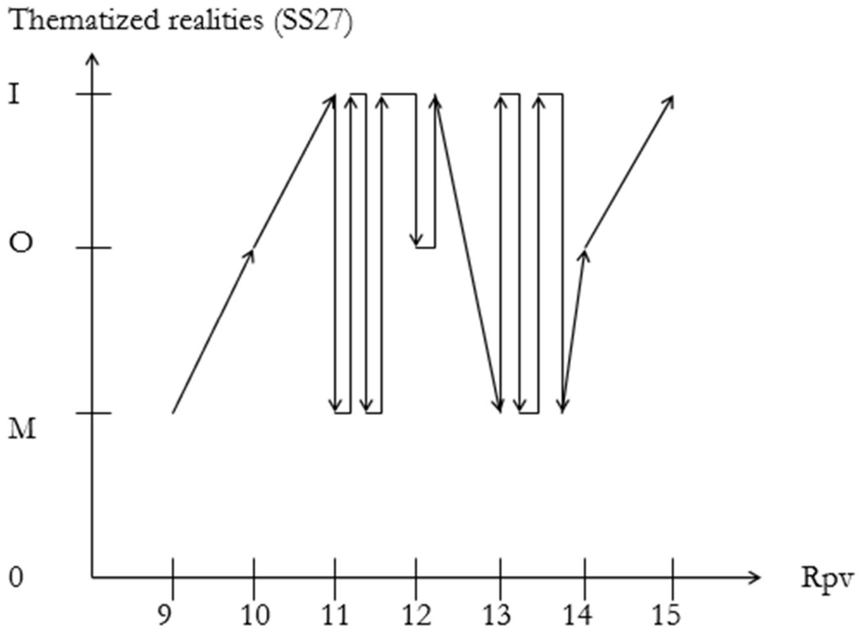
7 equivalence classes of P_i:

$$\begin{cases} \{O_1\} \\ \{O_2, O_4, O_{10}\} \\ \{O_3, O_5, O_7, O_{11}, O_{13}, O_{19}\} \\ \{O_6, O_8, O_{12}, O_{14}, O_{16}, O_{20}, O_{22}\} \\ \{O_9, O_{15}, O_{17}, O_{21}, O_{23}, O_{25}\} \\ \{O_{18}, O_{24}, O_{26}\} \\ \{O_{27}\} \end{cases}$$

3 equivalence classes of P_j:

$\{O_1, O_5, O_9, O_{11}, O_{13}, O_{21}, O_{25}\}$
$\{O_2, O_4, O_{10}, O_{14}, O_{18}, O_{24}, O_{26}\}$
$\{O_3, O_6, O_7, O_8, O_{12}, O_{15}, O_{16}, O_{17}, O_{19}, O_{20}, O_{22}, O_{23}, O_{27}\}$

Thus, in the attribute set P_i, the three objects in the second and sixth, the six objects in the third and fifth and the eight objects in the fourth equivalence classes, and in P_j, the seven objects in the first and second, and the thirteen objects in the third equivalence class are indiscernible. We may again visualize this in the following rough-fuzzy semiotic graph:



From this graph, in which we "unfolded" identical thematized realities in function of the same representation values, it clearly follows that graph(SS10) $\subset$ graph(SS27), i.e. that the rough-fuzzy set of SS10 is included in the rough-fuzzy set of SS27.

If we define now, as usual, the **P-lower** ($\underline{P}X$) and the **P-upper approximations** ($\bar{P}X$) of X:

$\underline{P}X = \{x \mid [x]_P \subseteq X\}$
$\bar{P}X = \{x \mid [x]_P \cap X \neq \varnothing\}$,

and the ordered pair $<\underline{P}X, \bar{P}X>$, as the rough set (whose elements may be fuzzy, cf. Komorowski, Polkowski, and Skowron 2000, pp. 46 ss.), then we get for target sets $X_{i,j} \subseteq \mathbf{U}$, whereby i indicates attribute set P_i and j indicates attribute set P_j:

For X_i = SS10: $<\underline{P}X_i, \bar{P}X_i> = \{O_1\} \cup \{O_2\} \cup \{O_3, O_4\} \cup \{O_5, O_7\} \cup \{O_6, O_8\} \cup \{O_9\} \cup \{O_{10}\}$

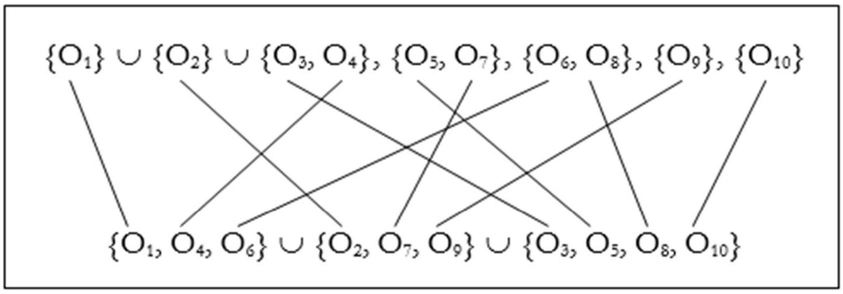
For X_j = SS10: $<\underline{P}X_j, \bar{P}X_j> = \{O_1, O_4, O_6\} \cup \{O_2, O_7, O_9\} \cup \{O_3, O_5, O_8, O_{10}\}$

For X_i = SS27: $<\underline{P}X_i, \bar{P}X_i> = <\{O_1\} \cup \{O_2, O_4, O_{10}\} \cup \{O_3, O_5, O_7, O_{11}, O_{13}, O_{19}\} \cup \{O_6, O_8, O_{12}, O_{14}, O_{16}, O_{20}, O_{22}\} \cup \{O_9, O_{15}, O_{17}, O_{21}, O_{23}, O_{25}\} \cup \{O_{18}, O_{24}, O_{26}\}, \{O_{27}\}>$

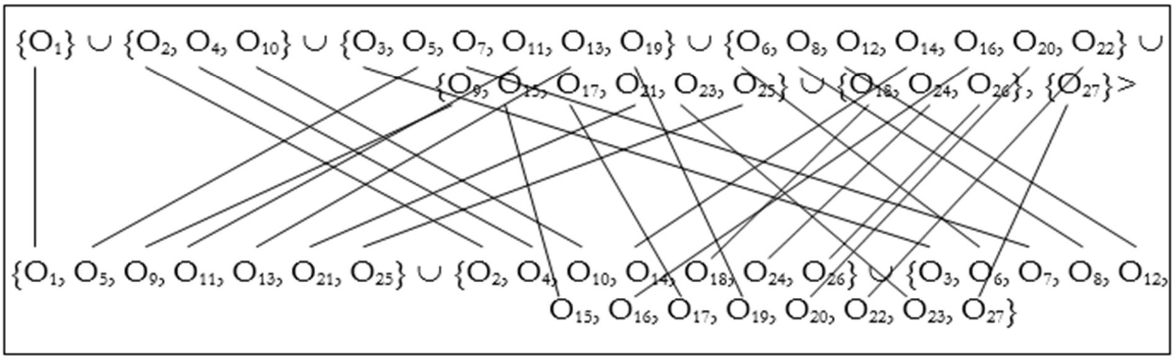
For X_j = SS27: $<\underline{P}X_j, \bar{P}X_j> = \{O_1, O_5, O_9, O_{11}, O_{13}, O_{21}, O_{25}\} \cup \{O_2, O_4, O_{10}, O_{14}, O_{18}, O_{24}, O_{26}\} \cup \{O_3, O_6, O_7, O_8, O_{12}, O_{15}, O_{16}, O_{17}, O_{19}, O_{20}, O_{22}, O_{23}, O_{27}\}$

If we have now a look at the four rough sets, we also recognize that in semiotics – at least as far as SS10 and SS27 are concerned – there is no way to construct reducts and cores from the sets of equivalences, and neither are there possibilities of feature extraction and construction of minimal sets of cuts (cf. Komorowski, Polkowski, and Skowron 2000, pp. 13ss.). However, the dependencies of the attributes of the sets P_i and P_j can be visualized as follows:

Dependencies of attributes in SS10:



Dependencies of attributes in SS27:



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Tetradic, triadic, and dyadic sign classes

1. In Toth (2008a, pp. 179 ss.), we have constructed a tetradic-tetratomic semiotics on the basis of the following 4×4 matrix:

	.0	.1	.2	.3
0.	0.0	0.1	0.2	0.3
1.	1.0	1.1	1.2	1.3
2.	2.0	2.1	2.2	2.3
3.	3.0	3.1	3.2	3.3

based on the general tetradic-tetratomic sign relation

SR_4 = R(Q, M, O, I); SR_4 = R(.0., .1., .2., .3.);

$SR_4 = (((Q \Rightarrow M) \Rightarrow O) \Rightarrow I)$; $SR_4 = (((.0. \Rightarrow .1.) \Rightarrow .2.) \Rightarrow .3.)$

with the tetratomic semiotic inclusion order

(3.a 2.b 1.c 0.d) with a, b, c, d $\in$ {.0., .1., .2., .3.} und $a \leq b \leq c \leq d$

We can then construct the following 35 tetradic-tetratomic sign classes and their dual reality thematics:

1	(3.0 2.0 1.0 0.0)	$\times$	(0.0 0.1 0.2 0.3)
2	(3.0 2.0 1.0 0.1)	$\times$	(1.0 0.1 0.2 0.3)
3	(3.0 2.0 1.0 0.2)	$\times$	(2.0 0.1 0.2 0.3)
4	(3.0 2.0 1.0 0.3)	$\times$	(3.0 0.1 0.2 0.3)
5	(3.0 2.0 1.1 0.1)	$\times$	(1.0 1.1 0.2 0.3)
6	(3.0 2.0 1.1 0.2)	$\times$	(2.0 1.1 0.2 0.3)
7	(3.0 2.0 1.1 0.3)	$\times$	(3.0 1.1 0.2 0.3)
8	(3.0 2.0 1.2 0.2)	$\times$	(2.0 2.1 0.2 0.3)
9	(3.0 2.0 1.2 0.3)	$\times$	(3.0 2.1 0.2 0.3)
10	(3.0 2.0 1.3 0.3)	$\times$	(3.0 3.1 0.2 0.3)
11	(3.0 2.1 1.1 0.1)	$\times$	(1.0 1.1 1.2 0.3)
12	(3.0 2.1 1.1 0.2)	$\times$	(2.0 1.1 1.2 0.3)

13 (3.0 2.1 1.1 0.3) × (3.0 1.1 1.2 0.3)

14 (3.0 2.1 1.2 0.2) × (2.0 2.1 1.2 0.3)

15 (3.0 2.1 1.2 0.3) × (3.0 2.1 1.2 0.3)

16 (3.0 2.1 1.3 0.3) × (3.0 3.1 1.2 0.3)

17 (3.0 2.2 1.2 0.2) × (2.0 2.1 2.2 0.3)

18 (3.0 2.2 1.2 0.3) × (3.0 2.1 2.2 0.3)

19 (3.0 2.2 1.3 0.3) × (3.0 3.1 2.2 0.3)

20 (3.0 2.3 1.3 0.3) × (3.0 3.1 3.2 0.3)

21 (3.1 2.1 1.1 0.1) × (1.0 1.1 1.2 1.3)

22 (3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3)

23 (3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3)

24 (3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)

25 (3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)

26 (3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)

27 (3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)

28 (3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)

29 (3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)

30 (3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)

31 (3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)

32 (3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)

33 (3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)

34 (3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)

35 (3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3)

The 35 representation systems can be ordered into the following system of **4 Tetratomic Tetrads of structural realities with dyadic thematization**:

1 (3.0 2.0 1.0 0.0) × ($\underline{\text{0.0 0.1 0.2 0.3}}$)

2 (3.0 2.0 1.0 0.1) × (1.0 $\underline{\text{0.1 0.2 0.3}}$)

3 (3.0 2.0 1.0 0.2) × (2.0 $\underline{0.1\ \ 0.2\ \ 0.3}$)

4 (3.0 2.0 1.0 0.3) × (3.0 $\underline{0.1\ \ 0.2\ \ 0.3}$)

11 (3.0 2.1 1.1 0.1) × ($\underline{1.0\ \ 1.1\ \ 1.2}$ 0.3)

21 (3.1 2.1 1.1 0.1) × ($\underline{1.0\ \ 1.1\ \ 1.2\ \ 1.3}$)

22 (3.1 2.1 1.1 0.2) × (2.0 $\underline{1.1\ \ 1.2\ \ 1.3}$)

23 (3.1 2.1 1.1 0.3) × (3.0 $\underline{1.1\ \ 1.2\ \ 1.3}$)

17 (3.0 2.2 1.2 0.2) × ($\underline{2.0\ \ 2.1\ \ 2.2}$ 0.3)

27 (3.1 2.2 1.2 0.2) × ($\underline{2.0\ \ 2.1\ \ 2.2}$ 1.3)

31 (3.2 2.2 1.2 0.2) × ($\underline{2.0\ \ 2.1\ \ 2.2\ \ 2.3}$)

32 (3.2 2.2 1.2 0.3) × (3.0 $\underline{2.1\ \ 2.2\ \ 2.3}$)

20 (3.0 2.3 1.3 0.3) × ($\underline{3.0\ \ 3.1\ \ 3.2}$ 0.3)

30 (3.1 2.3 1.3 0.3) × ($\underline{3.0\ \ 3.1\ \ 3.2}$ 1.3)

34 (3.2 2.3 1.3 0.3) × ($\underline{3.0\ \ 3.1\ \ 3.2}$ 2.3)

35 (3.3 2.3 1.3 0.3) × ($\underline{3.0\ \ 3.1\ \ 3.2\ \ 3.3}$)

Moreover, the 35 representation systems can also be ordered into the following system **of 4 Tetratomic Triads of triadic thematization**:

1 (3.0 2.0 1.0 0.0) × ($\underline{0.0\ \ 0.1\ \ 0.2\ \ 0.3}$)

6 (3.0 2.0 1.1 0.2) × (2.0 1.1 $\underline{0.2\ \ 0.3}$)

9 (3.0 2.0 1.2 0.3) × (3.0 2.1 $\underline{0.2\ \ 0.3}$)

7 (3.0 2.0 1.1 0.3) × (3.0 1.1 $\underline{0.2\ \ 0.3}$)

12 (3.0 2.1 1.1 0.2) × (2.0 $\underline{1.1\ \ 1.2}$ 0.3)

21 (3.1 2.1 1.1 0.1) × ($\underline{1.0\ \ 1.1\ \ 1.2\ \ 1.3}$)

25 (3.1 2.1 1.2 0.3) × (3.0 2.1 $\underline{1.2\ \ 1.3}$)

13 (3.0 2.1 1.1 0.3) × (3.0 $\underline{1.1\ \ 1.2}$ 0.3)

14 (3.0 2.1 1.2 0.2) × (2.0 2.1 1.2 0.3)

28 (3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)

31 (3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)

18 (3.0 2.2 1.2 0.3) × (3.0 2.1 2.2 0.3)

16 (3.0 2.1 1.3 0.3) × (3.0 3.1 1.2 0.3)

29 (3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)

19 (3.0 2.2 1.3 0.3) × (3.0 3.1 2.2 0.3)

35 (3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3)

2. Triadic-trichotomic semiotics that is constructed by aid of the following 3 × 3 matrix:

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

on the basis of the general triadic-trichotomic sign relation

SR_3 = R(M, O, I); SR_3 = R(.1., .2., .3.);

SR_3 = ((M ⇒ O) ⇒ I); SR_3 = ((.1. ⇒ .2.) ⇒ .3.)

with the trichotomic semiotic inclusion order

(3.a 2.b 1.c) with a, b, c ∈ {.1., .2., .3.} und a ≤ b ≤ c

has the following 10 triadic-trichotomic sign classes and their dual reality thematics:

1 (3.1 2.1 1.1) × (1.1 1.2 1.3)
2 (3.1 2.1 1.2) × (2.1 1.2 1.3)

3 (3.1 2.1 1.3) × (3.1 1.2 1.3)

4 (3.1 2.2 1.2) × (2.1 2.2 1.3)

5 (3.1 2.2 1.3) × (3.1 2.2 1.3)

6 (3.1 2.3 1.3) × (3.1 3.2 1.3)

7 (3.2 2.2 1.2) × (2.1 2.2 2.3)

8 (3.2 2.2 1.3) × (3.1 2.2 2.3)

9 (3.2 2.3 1.3) × (3.1 3.2 2.3)

10 (3.3 2.3 1.3) × (3.1 3.2 3.3)

The 10 representation systems can be ordered into the following system of **3 Trichotomic Triads** (Walther 1981, 1982):

1 (3.1 2.1 1.1) × ($\underline{1.1\ \ 1.2\ \ 1.3}$)

2 (3.1 2.1 1.2) × (2.1 $\underline{1.2\ \ 1.3}$)

3 (3.1 2.1 1.3) × (3.1 $\underline{1.2\ \ 1.3}$)

4 (3.1 2.2 1.2) × ($\underline{2.1\ \ 2.2}$ 1.3)

7 (3.2 2.2 1.2) × ($\underline{2.1\ \ 2.2\ \ 2.3}$)

8 (3.2 2.2 1.3) × (3.1 $\underline{2.2\ \ 2.3}$)

6 (3.1 2.3 1.3) × ($\underline{3.1\ \ 3.2}$ 1.3)

9 (3.2 2.3 1.3) × ($\underline{3.1\ \ 3.2}$ 2.3)

10 (3.3 2.3 1.3) × ($\underline{3.1\ \ 3.2\ \ 3.3}$)

Here, the dual-invariant sign class (3.1 2.2 1.3) × (3.1 2.2 1.3), the determinant of the triadic-trichotomic matrix, determines the system of the Trichotomic Triads. In the 2 systems of the 35 tetradic sign classes, the dual-invariant sign class (3.0 2.1 1.2 0.3) × (3.0 2.1 1.2 0.3), the determinant of the tetradic-tetratomic matrix, determines the 2 systems of the Tetratomic Tetrads. While (3.1 2.2 1.3) has the following three types of thematizations and thus structural realities:

$$(3.1\ 2.2\ 1.3) \times (\underline{3.1}\ \underline{2.1}\ \underline{1.3}) \rightarrow \begin{cases} (3.1, 2.1)\text{-them. } (1.3) \\ (3.1, 1.3)\text{-them. } (2.2) \\ (2.2, 1.3)\text{-them. } (3.1), \end{cases}$$

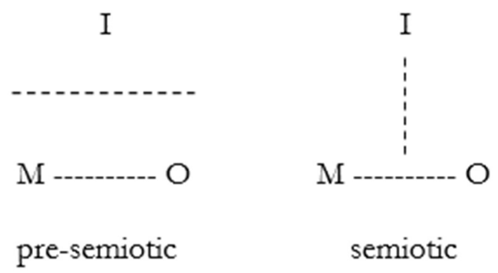
the sign class (3.0 2.1 1.2 0.3) has 10 types of thematizations and structural realities (thematized realities are underlined):

$$(3.0\ 2.1\ 1.2\ 0.3) \times (\underline{3.0}\ \underline{2.1}\ \underline{1.2}\ 0.3) \quad \rightarrow \quad \begin{array}{l} (\underline{3.0\ \ 2.1\ \ 1.2}\ \ 0.3) \\ (\underline{3.0\ \ 2.1}\ \ 1.2\ \ 0.3) \\ (\underline{3.0}\ \ 2.1\ \ 1.2\ \ 0.3) \\ (3.0\ \ \underline{2.1\ \ 1.2\ \ 0.3}) \\ (3.0\ \ 2.1\ \ \underline{1.2\ \ 0.3}) \\ (3.0\ \ 2.1\ \ 1.2\ \ \underline{0.3}) \\ (\underline{3.0}\ \ 2.1\ \ 1.2\ \ \underline{0.3}) \\ (3.0\ \ \underline{2.1\ \ 1.2}\ \ 0.3) \\ (3.0\ \ \underline{2.1}\ \ 1.2\ \ 0.3) \\ (3.0\ \ 2.1\ \ \underline{1.2}\ \ 0.3) \end{array}$$

Thus, from their structural realities and from their possibilities to be ordered into a system of n-atomic n-ads, SR_3 is **not** a part of SR_4, since SR_4 has quite different n-adic n-atomic and thematization structures than SR_3.

3. Ditterich (1990, pp. 29, 81) has defined the dyadic sign relation of de Saussure, which he calls „pre-semiotic", by aid of the semiotic matrix as a sub-relation of the triadic-trichotomic Peircean sign relation SR_3:

	.1	.2	.3
3.	3.1	3.2	3.3
2.	2.1	2.2	2.3
1.	1.1	1.2	1.3



$(1 \rightarrow 2) \ / \rightarrow 3 \qquad ((1 \rightarrow 2). \rightarrow 3)$

If we write the dyadic sign relation as SR_2, then we have according to Ditterich:

$SR_2 \subset SR_3$,

However, it is not clear, if this inclusion holds beyond the pure quantitative point of view. In the triadic sign model, the third category, the interpretant or the thirdness, alone guarantees that the triadic sign is a "mediating function between World and Consiousness" (Bense 1975, p. 16; 1976, p. 91; Toth 2008b). Thus, if the interpretant relation falls off, the sign cannot mediate anymore between the dyadic rest-function and the consciousness of the interpreter. Therefore, the interpretant relation which embeds the dyadic relation ($M \Rightarrow O$) into the triadic relation (($M \Rightarrow O) \Rightarrow I$) crosses the contexture of the denomination function ($M \Rightarrow O$) that belongs to the "world" and adds to it the designation function ($O \Rightarrow I$) that belongs to the "consciousness". Hence, already the triadic sign relation involves two logical contextures, world and consciousness, or object and subject that are bridged in the triadic sign relation. From that it follows, that Ditterich’s inclusion relation does not hold from the qualitative point of view (cf. also Toth 1991), so that we have

$SR_2 \not\subset SR_3$.

4. In Toth (2008c), I have introduced the tetradic-trichotomic pre-semiotic sign relation

PSR = (0., .1., .2., .3.); $SR_{4,3}$ (3.a 2.b 1.c 0.d)

with the corresponding trichotomic inclusion order

$(a \leq b \leq c)$,

whose corresponding semiotic structure is thus 4-adic, but 3-ary, since in Zr_k, the categorial number $k \neq 0$ (Bense 1975, p. 65), and therefore the pre-semiotic matrix is “defective” from the viewpoint of a quadratic matrix of Cartesian products over (.0., .1., .2., .3.):

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

From this semiotic matrix, we can construct the following 15 tetradic-trichotomic sign classes and their dual reality thematics:

1	$(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$
2	$(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$
3	$(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$
4	$(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$
5	$(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$
6	$(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$
7	$(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$
8	$(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$
9	$(3.1\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ 2.2\ 1.3)$
10	$(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$
11	$(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$
12	$(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2\ 2.3})$
13	$(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$
14	$(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$
15	$(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$,

whose number corresponds to the 15 trito-numbers of the polycontextural contexture T_4 (cf. Kronthaler 1986, p. 34), which underlines the fact that these 15 pre-semiotic sign classes are both quantitative and qualitative sign classes, because the integration of the zeroness into the triadic sign relation bridges the polycontextural border between the ontological space of objects and the semiotic space of signs (cf. Bense 1975, p. 65; Toth 2003).

Moreover, we notice that $SR_{4,3}$, unlike the systems SR_3 and SR_4, does not have a dual-identical sign class. On the other side, $SR_{4,3}$ displays, in the system of its dual reality thematics, semiotic structures that do neither occur in SR_3 nor in SR_4. Finally, in $SR_{4,3}$, we do not get any type of n-atomic n-ads, but the following system of **3 tetradic pentatomies** to which the 15 pre-semiotic sign classes can be ordered:

3 $(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$

4 $(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$

6 $(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$

7 $(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$

7 $(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$

11 $(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$

3 $(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$

6 $(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$

10 $(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$

9 $(3.1\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ 2.2\ 1.3)$

15 $(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$

12 $(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2\ 2.3})$

13 $(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$

14 $(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$

8 $(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$

5. As it was shown in Toth (2008c, d),

$SR_{4,3} \not\subset SR_4$,

since the category of zeroness appears only as tetradic, not as trichotomic semiotic value. Moreover, since zeroness (0.) or qualitiy (Q) localizes SR_3 in the ontological space (Bense 1975, p. 65), we also have

$SR_3 \not\subset SR_{4,3}$,

so that, by transitivity,

$SR_3 \not\subset SR_{4,3} \not\subset SR_4$,

and since we found above that

$SR_2 \not\subset SR_3$,

we finally obtain

$SR_2 \not\subset SR_3 \not\subset SR_{4,3} \not\subset SR_4$,

which means that the dyadic Saussurean sign relation is not a sub-relation of the triadic-trichotomic Peircean sign relation, the Peircean sign relation is not a sub-relation of the tetradic-trichotomic) pre-semiotic sign relation, and the latter is not a sub-relation of the tetradic-tetratomic sign relation, either!

However, it is true, from an exclusively quantitative standpoint, that we can visualize an "inclusion" relation between the four sign relations in the following semiotic matrix:

	.0	.1	.2	.3
0.	0.0	0.1	0.2	0.3
1.	1.0	1.1	1.2	1.3
2.	2.0	2.1	2.2	2.3
3.	3.0	3.1	3.2	3.3,

but in doing so, we ultimately "monocontexturalize" all higher semiotic relations down to the dyadic Saussurean "sign relation", which is not even a sign relation, but a dyadic sub-relation, namely the denomination relation of the complete triadic sign relation. Since the Saussurean sign relation corresponds exactly to the semiotic status of numbers in monocontextural mathematics, the following two systems of monocontexturalization of the four sign relations

(I) $SR_4 \rightarrow SR_3 \rightarrow SR_2$

(II) $SR_{4,3} \rightarrow SR_3 \rightarrow SR_2$

correspond to the reversal of fiberings from the system of Peano numbers into the system of polycontextural numbers (cf. Kronthaler 1986, pp. 93 s.). However, in semiotics, we have two different levels of semiotic monocontexturalization: In (I), the monocontexturalization goes strictly over the abolishment of categories, in $SR_3 \rightarrow SR_2$, the abolisment of the category of thirdness breaks down the "bridge" between world and consciousness or object and subject and turns the triadic sign relation into an "unsaturated" or "partial" sub-sign relation (Bense 1975, p. 44). Such a "sign relation" is thus beneath the recognition of a polycontextural border between sign and object, and this "sign relation" therefore cannot mediate between them. In (II), the monocontexturalization $SR_{4,3} \rightarrow SR_3$ abolishes the quality of zeroness and thus the qualitative embedding of SR_3; with the loss of this strictly qualitative category, the sign relation cannot mediate anymore between the levels of keno- and morphogrammatics on the one side, and semiotics on the other side, thus the polycontextural border between semiotic and ontological space (Bense 1975, p. 65) is abolished.

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Tetradic sign classes from relational and categorial numbers

1. In Toth (2008b), we had elaborated Bense’s introduction of relational and categorial numbers in order to fully characterize sign relations Zr_k (Bense 1975, pp. 65 s.). Zr_k includes pre-semiotic media relations (M°) which connect Zr_k as a representation scheme of the semiotic space with the ontological space out of which objects are selected to be thetically introduced as meta-objects and thus as signs (Bense 1967, p. 9). This distinction allows to differentiating between the semiotic sign relation

SR = (.1., .2., .3.)

and the pre-semiotic qualitative-quantitative sign relation

PSR = (0., .1., .2., .3.).

Since, in Zr_k, $k \neq 0$, the respective pre-semiotic matrix does not contain the zeroness in trichotomic position. Hence the pre-semiotic matrix is “defective” from the viewpoint of a total-symmetric matrix of Cartesian products over (.0., .1., .2., .3.):

	.1	.2	.3
0.	0.1	0.2	0.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

From that it follows, too, that sign classes built from the 12 sub-signs in the pre-semiotic matrix will not lead to the system of the 35 tetradic-tetratomic sign classes shown and discussed in Toth (2008a, pp. 179 ss.). If we apply the trichotomic semiotic order in triadic semiotic sign classes:

(3.a 2.b 1.c) with $a \leq b \leq c$

to the tetratomic order in tetradic pre-semiotic sign classes:

(3.a 2.b 1.c 0.d) with $a \leq b \leq c \leq d$,

then we can construct the following 15 pre-semiotic sign classes:

16 $\quad$ $(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$
17 $\quad$ $(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$
18 $\quad$ $(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$
19 $\quad$ $(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$
20 $\quad$ $(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$

21 $(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$
22 $(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$
23 $(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$
24 $(3.1\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ 2.2\ 1.3)$
25 $(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$
26 $(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$
27 $(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2\ 2.3})$
28 $(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$
29 $(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$
30 $(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$,

whose number corresponds to the 15 trito-numbers of the polycontextural contexture T_4 (cf. Kronthaler 1986, p. 34), which underlines the fact that these 15 pre-semiotic sign classes are both quantitative and qualitative sign classes, because the integration of the zeroness into the triadic sign relation bridges the polycontextural border between the ontological space of objects and the semiotic space of signs (cf. Toth 2003, 2008a).

Moreover, we notice that in the system of the 15 pre-semiotic classes, there is, unlike in the system of the 10 semiotic sign classes, no dual-identical sign class corresponding to the triadic "eigenreal" sign class (3.1 2.2 1.3) × (3.1 2.2 1.3), cf. Bense (1992). On the other side, the system of the 15 pre-semiotic sign classes displays, in the system of their dual reality thematics, semiotic structures that do not occur in the system of the 10 semiotic sign classes. In order to "formalize" them, we use the notational system introduced in Toth (2008a, pp. 176 ss.). The abbreviation HOM stands for homogeneous thematizations, LEFT and RIGHT refer to the direction of thematizations (indicated by arrows), and SWCH for "sandwich thematization" points to the fact that in the respective structural realities two realities are thematizing and two are thematized. Then we get the following types of tetradic thematizations of the 15 pre-semiotic sign classes:

1. Homogeneous thematizations:

1	$(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$	1^4	HOM
11	$(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$	2^4	HOM
15	$(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$	3^4	HOM

2. Dyadic thematizations

2.1. Dyadic-leftward thematizations

2	$(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$	$2^1 \leftarrow 1^3$	LEFT
3	$(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$	$3^1 \leftarrow 1^3$	LEFT
12	$(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2\ 2.3})$	$3^1 \leftarrow 2^3$	LEFT

2.2. Dyadic-rightward thematizations

7	$(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$	$2^3 \rightarrow 1^1$	RIGHT
10	$(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$	$3^3 \rightarrow 1^1$	RIGHT
14	$(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$	$3^3 \rightarrow 2^1$	RIGHT

2.3. Sandwich-Thematizations (only centripetal)

4	$(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$	$2^2 \leftrightarrow 1^2$	SWCH
6	$(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$	$3^2 \leftrightarrow 1^2$	SWCH
13	$(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$	$3^2 \leftrightarrow 2^2$	SWCH

3. Triadic thematizations

3.1. Triadic-leftward thematization

5	$(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$	$3^1 \leftrightarrow 2^1 \leftarrow 1^2$	LEFT

3.2. Triadic-rightward thematization

9	$(3.1\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ 2.2\ 1.3)$	$3^2 \rightarrow 2^1 \leftrightarrow 1^3$	RIGHT

3.3. Sandwich-thematization (centrifugal)

8	$(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$	$3^1 \leftarrow 2^2 \rightarrow 1^1$	SWCH

It is easy to recognize that the 15 reality thematics of the system of the tetradic pre-semiotic sign classes can not be organized into a system of tetratomic tetrads analogous to the system of trichotomic triads (cf. Walther 1982). The latter is symmetric by aid of the determinant of the eigenreal sign class (3.1 2.2 1.3), and since there is no eigenreality in the system of the 15 pre-semiotic sign classes, they can not be constructed as n-adic m-ary semiotic systems in which n = m like in the case of the tetratomic tetrads constructed out of the 35 tetradic-tetratomic sign classes in Toth (2008a, pp. 180 ss.).

However, it is possible to construct a system of **triadic pentatomies** out of the system of the 15 pre-semiotic sign classes:

1	$(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$	1^4	HOM
2	$(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$	$2^1 \leftarrow 1^3$	LEFT
4	$(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$	$2^2 \leftrightarrow 1^2$	SWCH
7	$(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$	$2^3 \rightarrow 1^1$	RIGHT
5	$(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$	$3^1 \leftrightarrow 2^1 \leftarrow 1^2$	LEFT

11	$(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$	2^4	HOM
3	$(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$	$3^1 \leftarrow 1^3$	LEFT
6	$(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$	$3^2 \leftrightarrow 1^2$	SWCH
10	$(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$	$3^3 \rightarrow 1^1$	RIGHT
9	$(3.1\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ 2.2\ 1.3)$	$3^2 \rightarrow 2^1 \leftrightarrow 1^3$	RIGHT

15	$(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$	3^4	HOM
12	$(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2\ 2.3})$	$3^1 \leftarrow 2^3$	LEFT
13	$(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$	$3^2 \leftrightarrow 2^2$	SWCH
14	$(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$	$3^3 \rightarrow 2^1$	RIGHT
8	$(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2}\ 1.3)$	$3^1 \leftarrow 2^2 \rightarrow 1^1$	SWCH

We recognize that each of these pentatomies has the following structure. X, Y, and Z $\in$ {1, 2, 3}:

X^4	HOM
$X^1 \leftarrow Y^3$	LEFT
$X^2 \leftrightarrow Y^2$	SWCH
$X^3 \rightarrow Y^1$	RIGHT
$X^1 \leftarrow X^2 \rightarrow Z^1$	SWCH

Thus, although the structural realities presented in the tetratomic reality thematics are tetradic, zeroness appears as triadic sign value and thus in the sign classes, but not as tetradic value and thus not in the reality thematics. In other words: In order to describe the realities presented by the tetradic pre-semiotic sign classes, **three** semiotic categories (X, Y, Z) are sufficient. Therefore, according to Bense (1975, pp. 64 ss. and Toth 2008b), the X, Y, Z refer to the **categorial numbers**, and the "exponents" in the above frequency notation of structural realities refer to the **relational numbers**. Using this frequency notation, we are able, on the basis of the above pentatomic structure of tetradic realities, to construct the system of the triadic pentatomies from the system of the 15 pre-semiotic sign classes based on the pre-semiotic sign relation PSR = (3.a 2.b 1.c 0.d), the tetratomic pre-semiotic order ($a \leq b \leq c \leq d$) and the restriction that zeroness must not appear in trichotomic position.

This n-adic m-ary semiotic system for n = 3 and m = 5 thus connects by its n-adic value the pre-semiotic system of the 15 sign classes with the triadic system of the 10 sign classes which therefore appear as a morphogrammatic fragment of the system of the 15 pre-semiotic sign classes, on the one side, and with a pentadic-m-ary system of $\leq$ 126 sign classes (cf. Toth 2008a, pp. 186 ss.) whose fragment the system of the 15 pre-semiotic sign classes is, on the other side (cf. Toth 2003, pp. 54 ss.).

Finally, one should notice that the absence of a dual-identical sign class in order to express eigenreality in the system of the 15 pre-semiotic sign classes leads to the fact that these pre-semiotic sign classes cannot be dualized, but must be triadized (cf. Kronthaler 1992, p. 293). Triadization is thus the minimal condition to transform one of the 15 pre-semiotic sign classes by reversing both the order of its dyadic sub-relations and of its monadic prime-signs back to its original sign class structure:

7 $(3.1\ 2.2\ 1.2\ 0.3) \times (3.0\ 2.1\ 2.2\ 1.3) \times (3.1\ 2.2\ 1.2\ 0.3)$,

The present study is the first contribution to **Pre-semiotics** in the sense of the theory of the pre-semiotic sign classes, their **trial** reality thematics and their associated system of triadic pentatomies. The main aim of Pre-semiotics is to formally analyze and describe the "never-land" between the Ontological and the Semiotic Space in the sense of Bense (1975, p. 65) and to disclose the pre-semiotic relations in the network of the abyss between sign and object.

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Tetratomic tetrads from an extension of the set of the pre-semiotic sign classes

1. Unlike the "classic" semiotic sign relation $SR_{3,3}$ = (3.a 2.b 1.c), which is triadic-trichotomic, the "trans-classic" pre-semiotic sign relation $SR_{4,3}$ = (3.a 2.b 1.c 0.d) is tetradic-trichotomic. As a tetradic-trichotomic sign relation, $SR_{4,3}$ thus can be considered an expansion of $SR_{3,3}$. However, at the same time, $SR_{4,3}$ is also a fragment of the tetradic-tetratomic sign relation $SR_{4,4}$ (cf. Toth 2007, pp. 214 ss.), which can be seen best if we have a look at the structural realities presented by the reality thematics of the 15 pre-semiotic sign classes:

31 (3.1 2.1 1.1 0.1) × (1.0 $\underline{1.1\ 1.2}$ 1.3)
32 (3.1 2.1 1.1 0.2) × (2.0 $\underline{1.1\ 1.2}$ 1.3)
33 (3.1 2.1 1.1 0.3) × (3.0 $\underline{1.1\ 1.2}$ 1.3)

34 (3.1 2.1 1.2 0.2) × (2.0 $\underline{2.1\ 1.2}$ 1.3)
35 (3.1 2.1 1.2 0.3) × (3.0 $\underline{2.1\ 1.2}$ 1.3)

36 (3.1 2.1 1.3 0.3) × (3.0 $\underline{3.1\ 1.2}$ 1.3)

37 (3.1 2.2 1.2 0.2) × (2.0 $\underline{2.1\ 2.2}$ 1.3)
38 (3.1 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2}$ 1.3)

39 (3.1 2.2 1.3 0.3) × (3.0 $\underline{3.1\ 2.2}$ 1.3)

40 (3.1 2.3 1.3 0.3) × (3.0 $\underline{3.1\ 3.2}$ 1.3)

41 (3.2 2.2 1.2 0.2) × (2.0 $\underline{2.1\ 2.2}$ 2.3)
42 (3.2 2.2 1.2 0.3) × (3.0 $\underline{2.1\ 2.2}$ 2.3)

43 (3.2 2.2 1.3 0.3) × (3.0 $\underline{3.1\ 2.2}$ 2.3)

44 (3.2 2.3 1.3 0.3) × (3.0 $\underline{3.1\ 3.2}$ 2.3)
45 (3.3 2.3 1.3 0.3) × (3.0 $\underline{3.1\ 3.2}$ 3.3)

Thus, the reality thematic of the first trichotomic triad is characterized by (1.1 1.2). It turns out that each of the 15 reality thematics can be embedded into a trichotomic triads characterized by a pair of sub-signs. However, in order to do that, we have to reconstruct a semiotic system whose part SS15 is. As one easily sees, it is not SS35, which is built from the tetradic-tetratomic sign relation $SR_{4,4}$ = (3.a 2.b 1.c 0.d) and the semiotic inclusion order $a \leq b \leq c \leq d$, since in the pre-semiotic system SS15, a, b, c, d $\in$ {1, 2, 3} and thus $\neq 0$ (cf. Bense 1975, p. 65; Toth 2008a).

In the following table, we reconstruct the lacking reality thematics to build trichotomic triads by asterisk (*, **):

1 (3.1 2.1 1.1 0.1) × (1.0 $\underline{1.1\ 1.2}$ 1.3)
2 (3.1 2.1 1.1 0.2) × (2.0 $\underline{1.1\ 1.2}$ 1.3)
3 (3.1 2.1 1.1 0.3) × (3.0 $\underline{1.1\ 1.2}$ 1.3)

*	(3.1 2.1 1.2 0.1) × (1.0 2.1 1.2 1.3)
4	(3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)
5	(3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)
*	(3.1 2.1 1.3 0.1) × (1.0 3.1 1.2 1.3)
**	(3.1 2.1 1.3 0.2) × (2.0 3.1 1.2 1.3)
6	(3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)
*	(3.1 2.2 1.2 0.1) × (1.0 2.1 2.2 1.3)
7	(3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)
8	(3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)
*	(3.1 2.2 1.3 0.1) × (1.0 3.1 2.2 1.3)
**	(3.1 2.2 1.3 0.2) × (2.0 3.1 2.2 1.3)
9	(3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)
*	(3.1 2.3 1.3 0.1) × (1.0 3.1 3.2 1.3)
**	(3.1 2.3 1.3 0.2) × (2.0 3.1 3.2 1.3)
10	(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)
*	(3.2 2.2 1.2 0.1) × (1.0 2.1 2.2 2.3)
11	(3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)
12	(3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)
*	(3.2 2.2 1.3 0.1) × (1.0 3.1 2.2 2.3)
**	(3.2 2.2 1.3 0.2) × (2.0 3.1 2.2 2.3)
13	(3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)
*	(3.2 2.3 1.3 0.1) × (1.0 3.1 3.2 2.3)
**	(3.2 2.3 1.3 0.2) × (2.0 3.1 3.2 2.3)
14	(3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)
*	(3.3 2.3 1.3 0.1) × (1.0 3.1 3.2 3.3)
**	(3.3 2.3 1.3 0.2) × (2.0 3.1 3.2 3.3)
15	(3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3)

2. Hence, we get 10 trichotomic triads and thus a system of 30 pre-semiotic sign classes (SS30). However, the set SS30\SS15 contains sign classes that are not built according to the inclusion order (a ≤ b ≤ c ≤ d), which is valid for SS15. But note that this "violation" of semiotic inclusion touches only trichotomic zeroness, i.e. d, so that SS30 can be characterized by the following pre-semiotic inclusion orders:

a ≤ b ≤ c < d, e.g. (3.1 2.1 1.2 0.3)
a ≤ b ≤ c = d, e.g. (3.1 2.1 1.2 0.2)
a ≤ b ≤ c > d, e.g. (3.1 2.1 1.2 0.1)

Without this constraint that is based on Bense's distinction between relational and categorial numbers (cf. Toth 2008a), the maximal amount of sign classes from $SR_{4,3}$ would be 43 = 64.

Moreover, if we look, e.g. at the reality thematic of the following pre-semiotic dual system:

13 $(3.2\ 2.2\ 1.3\ 0.3) \times (3.0\ \underline{3.1\ 2.2}\ 2.3)$

we recognize that the pair of sub-signs characteristic for the embedding of no. 13 into a trichotomic triads (3.1 2.2) belongs partly to the thematizing and partly to the thematized group of sub-signs in the following structural reality:

$(\underline{3.0\ 3.1}\ \underline{2.2\ 2.3}) \equiv 32 \leftrightarrow 22$,

which thus can be interpreted both as object-thematized interpretant ($32 \leftarrow 22$) and as interpretant-thematized object ($32 \rightarrow 22$).

3. We will now order the 30 pre-semiotic sign classes over this extension of $SR_{4,3}$, which we shall call $SR_{4,3}*$, according to their types of thematizations introduced in Toth (2007, pp. 214 ss.).

1. Homogeneous thematizations:

1	$(3.1\ 2.1\ 1.1\ 0.1) \times (\underline{1.0\ 1.1\ 1.2\ 1.3})$	14
11	$(3.2\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2\ 2.3})$	24
15	$(3.3\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2\ 3.3})$	34

2. Dyadic thematizations:

2	$(3.1\ 2.1\ 1.1\ 0.2) \times (2.0\ \underline{1.1\ 1.2\ 1.3})$	$21 \leftarrow 13$
3	$(3.1\ 2.1\ 1.1\ 0.3) \times (3.0\ \underline{1.1\ 1.2\ 1.3})$	$31 \leftarrow 13$
4	$(3.1\ 2.1\ 1.2\ 0.2) \times (\underline{2.0\ 2.1}\ \underline{1.2\ 1.3})$	$22 \leftrightarrow 12$
6	$(3.1\ 2.1\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{1.2\ 1.3})$	$32 \leftrightarrow 12$
7	$(3.1\ 2.2\ 1.2\ 0.2) \times (\underline{2.0\ 2.1\ 2.2}\ 1.3)$	$23 \rightarrow 11$
10	$(3.1\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 1.3)$	$33 \rightarrow 11$
*	$(3.2\ 2.2\ 1.2\ 0.1) \times (1.0\ \underline{2.1\ 2.2\ 2.3})$	$11 \leftarrow 23$
12	$(3.2\ 2.2\ 1.2\ 0.3) \times (3.0\ \underline{2.1\ 2.2\ 2.3})$	$31 \leftarrow 23$
13	$(3.2\ 2.2\ 1.3\ 0.3) \times (\underline{3.0\ 3.1}\ \underline{2.2\ 2.3})$	$32 \leftrightarrow 22$
14	$(3.2\ 2.3\ 1.3\ 0.3) \times (\underline{3.0\ 3.1\ 3.2}\ 2.3)$	$33 \rightarrow 21$
*	$(3.3\ 2.3\ 1.3\ 0.1) \times (1.0\ \underline{3.1\ 3.2\ 3.3})$	$11 \leftarrow 33$
*	$(3.3\ 2.3\ 1.3\ 0.2) \times (2.0\ \underline{3.1\ 3.2\ 3.3})$	$21 \leftarrow 33$

3. Triadic thematizations:

*	$(3.1\ 2.1\ 1.2\ 0.1) \times (1.0\ 2.1\ \underline{1.2\ 1.3})$	$11 \leftrightarrow 21 \leftarrow 12$
5	$(3.1\ 2.1\ 1.2\ 0.3) \times (3.0\ 2.1\ \underline{1.2\ 1.3})$	$31 \leftrightarrow 21 \leftarrow 12$
*	$(3.1\ 2.1\ 1.3\ 0.1) \times (1.0\ 3.1\ \underline{1.2\ 1.3})$	$11 \leftrightarrow 31 \leftarrow 12$
*	$(3.1\ 2.1\ 1.3\ 0.2) \times (2.0\ 3.1\ \underline{1.2\ 1.3})$	$21 \leftrightarrow 31 \leftarrow 12$
*	$(3.1\ 2.2\ 1.2\ 0.1) \times (1.0\ \underline{2.1\ 2.2}\ 1.3)$	$11 \leftarrow 22 \rightarrow 11$

8	(3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)	31 ← 22 → 11
9	(3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)	32 → 21 ↔ 11
*	(3.1 2.3 1.3 0.1) × (1.0 3.1 3.2 1.3)	11 ← 32 → 11
*	(3.1 2.3 1.3 0.2) × (2.0 3.1 3.2 1.3)	21 ← 32 → 11
*	(3.2 2.2 1.3 0.1) × (1.0 3.1 2.2 2.3)	11 ↔ 31 ← 22
*	(3.2 2.2 1.3 0.2) × (2.0 3.1 2.2 2.3)	21 ↔ 31 ← 22
*	(3.2 2.3 1.3 0.1) × (1.0 3.1 3.2 2.3)	11 ← 32 → 21
*	(3.2 2.3 1.3 0.2) × (2.0 3.1 3.2 2.3)	21 ← 32 → 21

4. Tetradic thematizations:

*	(3.1 2.2 1.3 0.1) × (1.0 3.1 2.2 1.3)	11 ↔ 31 ↔ 21 ↔ 11
*	(3.1 2.2 1.3 0.2) × (2.0 3.1 2.2 1.3)	21 ↔ 31 ↔ 21 ↔ 11

We can now group these n-adic thematizations to tetratomic n-ads. It turns out that reality thematics, which present dyadic thematization, can be grouped into 3 tetratomic tetrads:

Tetratomic Tetrads of dyadic thematization

2	(3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3)	21 ← 13
3	(3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3)	31 ← 13
4	(3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)	22 ↔ 12
6	(3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)	32 ↔ 12
7	(3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)	23 → 11
10	(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)	33 → 11
*	(3.2 2.2 1.2 0.1) × (1.0 2.1 2.2 2.3)	11 ← 23
12	(3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)	31 ← 23
13	(3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)	32 ↔ 22
14	(3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)	33 → 21
*	(3.3 2.3 1.3 0.1) × (1.0 3.1 3.2 3.3)	11 ← 33
*	(3.3 2.3 1.3 0.2) × (2.0 3.1 3.2 3.3)	21 ← 33

Reality thematics, which present dyadic thematization, can be grouped into 3 tetratomic tetrads plus the $SR_{4,3}$-equivalent of the dual-identical sign class (3.1 2.2 1.3) in $SR_{3,3}$:

*	(3.1 2.1 1.2 0.1) × (1.0 2.1 1.2 1.3)	11 ↔ 21 ← 12
5	(3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)	31 ↔ 21 ← 12
*	(3.1 2.1 1.3 0.1) × (1.0 3.1 1.2 1.3)	11 ↔ 31 ← 12
*	(3.1 2.1 1.3 0.2) × (2.0 3.1 1.2 1.3)	21 ↔ 31 ← 12
*	(3.1 2.2 1.2 0.1) × (1.0 2.1 2.2 1.3)	11 ← 22 → 11
8	(3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)	31 ← 22 → 11
*	(3.1 2.3 1.3 0.1) × (1.0 3.1 3.2 1.3)	11 ← 32 → 11
*	(3.2 2.3 1.3 0.2) × (2.0 3.1 3.2 2.3)	21 ← 32 → 21

* (3.1 2.3 1.3 0.2) × (2.0 <u>3.1 3.2</u> 1.3) 21 ← 32 → 11
* (3.2 2.2 1.3 0.1) × (1.0 3.1 <u>2.2 2.3</u>) 11 ↔ 31 ← 22
* (3.2 2.2 1.3 0.2) × (2.0 3.1 <u>2.2 2.3</u>) 21 ↔ 31 ← 22
* (3.2 2.3 1.3 0.1) × (1.0 <u>3.1 3.2</u> 2.3) 11 ← 32 → 21

9 (3.1 2.2 1.3 0.3) × (<u>3.0 3.1</u> 2.2 1.3) 32 → 21 ↔ 11

Although the tetradic pre-semiotic sign class (3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3) is only dual-invariant respecting its triadic part relation (3.1 2.2 1.3), the sign class (3.1 2.2 1.3 0.3) and its reality thematic (3.0 3.1 2.2 1.3) hang together with all other sign classes and reality thematics of this tetratomic tetrad of triadic thematization, respectively, by at least one sub-sign. Thus, the pre-semiotic dual system (3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3) shares this type of connectedness with the semiotic dual system (3.1 2.2 1.3) × (3.1 2.2 1.3).

4. In Toth (2008b), we have shown that both the semiotic system SS10 over $SR_{3,3}$ and the pre-semiotic system SS27 over $SR_{3,3}$ with abolishment of the semiotic inclusion order $a \leq b \leq c$ are homeostatic. It thus may astonish that also both SS15 and SS30 over $SR_{4,3}$ are homeostatic, despite their lacking of a (genuine) dual-identical sign class. The reason is the for-mentioned connectedness of the pre-semiotic dual system (3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3) by at least one sub-sign to all other pre-semiotic dual systems both from SS15 and from SS30:

1	(3.1 2.1 1.1 0.1) × (1.0 1.1 1.2 1.3)
2	(3.1 2.1 1.1 0.2) × (2.0 1.1 1.2 1.3)
3	(3.1 2.1 1.1 0.3) × (3.0 1.1 1.2 1.3)
*	(3.1 2.1 1.2 0.1) × (1.0 2.1 1.2 1.3)
4	(3.1 2.1 1.2 0.2) × (2.0 2.1 1.2 1.3)
5	(3.1 2.1 1.2 0.3) × (3.0 2.1 1.2 1.3)
*	(3.1 2.1 1.3 0.1) × (1.0 3.1 1.2 1.3)
**	(3.1 2.1 1.3 0.2) × (2.0 3.1 1.2 1.3)
6	(3.1 2.1 1.3 0.3) × (3.0 3.1 1.2 1.3)
*	(3.1 2.2 1.2 0.1) × (1.0 2.1 2.2 1.3)
7	(3.1 2.2 1.2 0.2) × (2.0 2.1 2.2 1.3)
8	(3.1 2.2 1.2 0.3) × (3.0 2.1 2.2 1.3)
*	(3.1 2.2 1.3 0.1) × (1.0 3.1 2.2 1.3)
**	(3.1 2.2 1.3 0.2) × (2.0 3.1 2.2 1.3)
9	**(3.1 2.2 1.3 0.3) × (3.0 3.1 2.2 1.3)**
*	(3.1 2.3 1.3 0.1) × (1.0 3.1 3.2 1.3)
**	(3.1 2.3 1.3 0.2) × (2.0 3.1 3.2 1.3)
10	(3.1 2.3 1.3 0.3) × (3.0 3.1 3.2 1.3)
*	(3.2 2.2 1.2 0.1) × (1.0 2.1 2.2 2.3)
11	(3.2 2.2 1.2 0.2) × (2.0 2.1 2.2 2.3)
12	(3.2 2.2 1.2 0.3) × (3.0 2.1 2.2 2.3)
*	(3.2 2.2 1.3 0.1) × (1.0 3.1 2.2 2.3)
**	(3.2 2.2 1.3 0.2) × (2.0 3.1 2.2 2.3)
13	(3.2 2.2 1.3 0.3) × (3.0 3.1 2.2 2.3)
*	(3.2 2.3 1.3 0.1) × (1.0 3.1 3.2 2.3)
**	(3.2 2.3 1.3 0.2) × (2.0 3.1 3.2 2.3)
14	(3.2 2.3 1.3 0.3) × (3.0 3.1 3.2 2.3)
*	(3.3 2.3 1.3 0.1) × (1.0 3.1 3.2 3.3)
**	(3.3 2.3 1.3 0.2) × (2.0 3.1 3.2 3.3)
15	(3.3 2.3 1.3 0.3) × (3.0 3.1 3.2 3.3)

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The multiple reality notion in n-contextural semiotics

1. Each monocontextural sign class of the general abstract form

SCl = (3.a 2.b 1.c)

is bijectively mapped onto its dual reality thematic

$\times$(3.a 2.b 1.c) = (c.1 b.2 a.3)

in order to form a so-called semiotic dual system:

DS = (3.a 2.b 1.c) $\times$ (c.1 b.2 a.3).

2. However, in polycontextural semiotics there is not only one, but at least two possibilities for “dualization” and thus for reality thematics. From the abstract form of the 3-contextural sign class

SCl = ($3.a_{i,j}$ $2.b_{i,j}$ $1.c_{ij}$),

we can get

$\times_1(3.a_{i,j}\ 2.b_{i,j}\ 1.c_{ij}) = (c.1_{i,j}\ b.2_{i,j}\ a.3_{ij})$

$\times_2(3.a_{i,j}\ 2.b_{i,j}\ 1.c_{ij}) = (c.1_{j,i}\ b.2_{j,i}\ a.3_{j,i})$.

While the 3-contextural dual system

DS = $(3.a_{i,j}\ 2.b_{i,j}\ 1.c_{ij}) \times_1 (c.1_{i,j}\ b.2_{i,j}\ a.3_{ij})$

can be shown in one and the same semiotic matrix, e.g. for

DS = $(3.1_3\ 2.2_{1,2}\ 1.2_1) \times_1 (2.1_1\ 2.2_{1,2}\ 1.3_3)$

$$\begin{pmatrix} 1.1_{1,3} & 1.2_1 & 1.3_3 \\ 2.1_1 & 2.2_{1,2} & 2.3_2 \\ 3.1_3 & 3.2_2 & 3.3_{2,3} \end{pmatrix}$$

the dual system

$DS = (3.a_{i,j}\ 2.b_{i,j}\ 1.c_{ij}) \times_2 (c.1_{j,i}\ b.2_{j,i}\ a.3_{j,i})$

needs two semiotic matrices in order to be display, f.ex. for

$DS = (3.1_3\ 2.2_{1,2}\ 1.2_1) \times_2 (2.1_1\ 2.2_{2,1}\ 1.3_3)$

$$\begin{pmatrix} 1.1_{1,3} & 1.2_1 & 1.3_3 \\ 2.1_1 & 2.2_{1,2} & 2.3_2 \\ 3.1_3 & 3.2_2 & 3.3_{2,3} \end{pmatrix} \quad \begin{pmatrix} 1.1_{3,1} & 1.2_1 & 1.3_3 \\ 2.1_1 & 2.2_{2,1} & 2.3_2 \\ 3.1_3 & 3.2_2 & 3.3_{3,2} \end{pmatrix}$$

whereby the two matrices are chiral, i.e. there is no way to superimpose the mirror pictures.

3. If have now a look at the same sign class in 4-contextures, we get

$SCl = (3.1_{3,4}\ 2.2_{1,2,4}\ 1.2_{1,4})$

$\times_1(2.1_{1,4}\ 2.2_{1,2,4}\ 1.3_{3,4})$

$\times_2(2.1_{4,1}\ 2.2_{4,2,1}\ 1.3_{4,3})$

$\times_3(2.1_{4,1}\ 2.2_{1,4,2}\ 1.3_{4,3})$

$\times_4(2.1_{4,1}\ 2.2_{2,1,4}\ 1.3_{4,3})$

$\times_5(2.1_{4,1}\ 2.2_{2,4,1}\ 1.3_{4,3})$

$\times_6(2.1_{4,1}\ 2.2_{4,1,2}\ 1.3_{4,3})$

and thus 6 different "reality thematics" – and these are not all, since combinations have not been looked for here.

So, while for

$1\text{-}SCl = \times_1\times_1(3.1\ 2.2\ 1.2) = (3.1\ 2.2\ 1.2)$,

we have for n-contextural sign classes with n > 1

$3\text{-}SCl = \times_2\times_2\times_2(3.1_3\ 2.2_{1,2}\ 1.2_1) = (3.1_3\ 2.2_{1,2}\ 1.2_1)$

$4\text{-}SCl = \times_3\times_3\times_3\times_3(3.1_{3,4}\ 2.2_{1,2,4}\ 1.2_{1,4}) = (3.1_{3,4}\ 2.2_{1,2,4}\ 1.2_{1,4})$

Regarding reality, we thus have 1 thematized reality for 1-SCl, 2 thematized realities for 3-SCl, 6 thematized realities for 4-SCl, but only as long as all sign classes are triadic! Hence generally, every n-contextural 3-adic sign class has (n-1)! thematized realities, so that n-times application of $\times_n$ closes this "semiotic Hamilton circle". It should be clear, that from these considerations, it results, that there are neither 1 nor 10 (cf. Bense 1980) nor 15 nor 35, ..., but infinite semiotic realities.

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